

# VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ

BRNO UNIVERSITY OF TECHNOLOGY

FAKULTA ELEKTROTECHNIKY A KOMUNIKAČNÍCH TECHNOLOGIÍ  
ÚSTAV VÝKONOVÉ ELEKTROTECHNIKY A ELEKTRONIKY

FACULTY OF ELECTRICAL ENGINEERING AND COMMUNICATION

DEPARTMENT OF POWER ELECTRICAL AND ELECTRONIC ENGINEERING

## **Control of salient PM machine using d-q frame machine model and Matlab Simulink**

DIPLOMOVÁ PRÁCE

MASTER'S THESIS

AUTOR PRÁCE  
AUTHOR

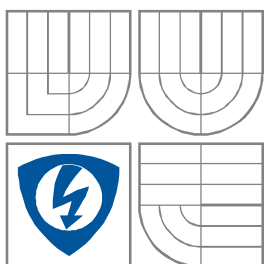
Bc. Jiří Dušek

BRNO 2010



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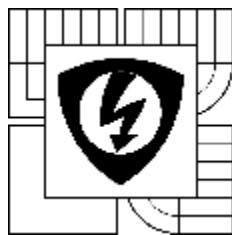
**Bc. Jiří Dušek**

**VEDOUCÍ PRÁCE**

SUPERVISOR

**doc. Ing. Čestmír Ondrůšek, CSc.**

**BRNO, 2010**



**BRNO UNIVERSITY  
OF TECHNOLOGY**

**Faculty of Electrical Engineering  
and Communication**

**Department of Power Electrical and Electronic  
Engineering**

# Diploma thesis

master's study field

**Power Electrical and Electronic Engineering**

**Student:** Bc. Jiří Dušek

**Year of study:** 2

**ID:** 78036

**Academic year:** 2009/10

## TITLE OF THESIS:

**Control of salient PM machine using d-q frame machine  
model and Matlab Simulink**

## INSTRUCTION:

1. Provide the literature search.
2. Design accurate torque control for wide torque and speed range.
3. Field weakening optimization in torque and speed range.
4. Efficiency calculation.
5. Evaluate the results.

## REFERENCE:

**Assignment deadline:** 1.10.2009

**Submission deadline:** 20.5.2010

**Head of thesis:** doc. Ing. Čestmír Ondrůšek, CSc.

**Consultant:**

**doc. Ing. Čestmír Ondrůšek, CSc.**  
*Subject Council chairman*

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## **Abstrakt**

Tato práce se zabývá synchronním motorem s permanentními magnety na rotoru (PMSM), jeho modelováním a návrhu regulační struktury. V práci jsou uvedeny způsoby a výhody použití permanentních magnetů v elektrických motorech. Dále se práce zabývá transformací třífázové soustavy do dq0. Pomocí Parkovy transformace jsou v práci odvozeny rovnice stroje v dq0 souřadnicovém systému a vytvořeny náhradní schémata stroje v dq osách. Rovnice i schémata zahrnují jak ztráty v mědi, tak ztráty v železe. Náhradní schémata jsou popsány elektrickými a mechanickými rovnicemi a následně překresleny do grafické podoby v programu Matlab Simulink. Vytvořeny jsou dva modely PMSM, jeden s uvažováním ztrát v železe a druhý bez těchto ztrát. Pro oba dva modely je zde popsán postup návrhu regulátorů proudu a otáček. Pro model, u kterého jsou uvažovány ztráty v železe je navíc použito více druhů řídicích strategií a tyto strategie jsou mezi sebou navzájem porovnány.

## **Abstract**

This thesis deals with permanent magnet synchronous machine (PMSM) and its modeling. There are mentioned ways of the advantages and use of permanent magnets in electric machines and machines which uses permanent magnets. The transformation of three-phase system to the dq0 reference frame using the Park's transformation is described and used for description of PMSM. The equations in dq0 reference frame and equivalent circuit of PMSM in dq0 system are established. These equations takes in consideration copper and iron losses.. Due to derived equations and circuit, PMSM is described and model in Matlab –Simulink is realized. For both of models of machines are designed currents and revolutions controllers. For model of PMSM where losses are taken in consideration is more kinds of control strategies discussed and used. These strategies are compared between each other in this thesis. Also results of simulations of different states are given.

## **Klíčová slova**

Synchronní motor s permanentními magnety; matematický model; dq0 souřadný systém; Matlab-Simulink; Parkova transformace, návrh regulátoru, řídicí strategie.

## **Keywords**

Permanent magnet synchronous machine, mathematical model, dq0 reference frame, Matlab-Simulink, Park's transformation, controller design, control strategies

## **Bibliographical citation**

Dušek, J. Control of salient PM machine using d-q frame machine model and Matlab Simulink, Brno: FEEC BUT in Brno, 2010. 61 p.

## Prohlášení

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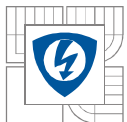
Podpis autora .....

## Acknowledgments

I would really like to say thanks to my family for support that they have given me during my life and my studies. Also I want to say thanks to my girlfriend, who has stayed with me across all difficulties that appears during writing this thesis. Last but not least I want to say huge thanks to my supervisor doc. Ing. Čestmír Ondrůšek, CSc. for his great advices and for all experiences which he gave me. Thank you all very, very much.

V Brně dne .....

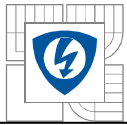
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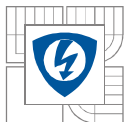


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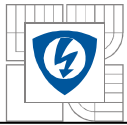
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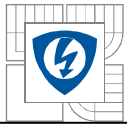
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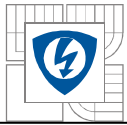


## LIST OF SHORTCUTS AND SYMBOLS

Shortcut of symbol	Definition	Unit
AC	Alternating current	[-]
CM	Control method	[-]
DC	Direct current	[-]
dq0	Direct-quadrature-zero	[-]
EMF	Electro-motive force	[V]
$F_{Ci}$	Transfer function of current controller	[-]
$F_{C\omega}$	Transfer function of revolutions controller	[-]
Fig.	Figure	[-]
$F_S$	Transfer function of	[-]
$F_{SL}$	Transfer function of system with losses	[-]
$F_{wi}$	Transfer function of	[-]
HEV	Hybrid electric vehicle	[-]
$I_a$	Armature current	[A]
$I_{am}$	Maximal possible $I_a$	[A]
$i_d, i_q$	d-, q-axis current respectively	[A]
$i_{d0}, i_{q0}$	Current in shunt branch to resistance represents core loss in d-, q-axis	[A]
$i_{dc}, i_{qc}$	Current through resistance $R_c$ in d-, q-axis	[A]
$i_{df}^*$	Wanted $i_d$ due to FW control strategy	[A]
$i_{dt}^*$	Wanted $i_d$ due to MTPA control strategy	[A]
$I_{nom}$	Rated rms line phase current	[A]
J	Moment of inertia of the entire rotor assembly	[kgm <sup>2</sup> ]
$k_d, k_q, k_0$	Transformation constants for direct-, quadrature-axis and zero sequence	[-]
$K_{FI}$	Frequency inverter constant	[-]
$L_d, L_q$	d- and q-axis inductances respectively	[mH]
ME	Maximum efficiency	[-]
mmf	Magneto-motive force	[A]
MTPA	Maximal torque per ampere	[-]
p	Laplace operator	[-]
P	Output power	[W]
$P_{Cu}$	Copper losses	[W]
$P_E$	Electrical losses	[W]
$P_{Fe}$	Iron losses	[W]
$P_L$	Total losses	[W]
PM	Permanent magnet	[-]
$P_M$	Mechanical losses	[W]
PMSM	Permanent magnet synchronous machine	[-]



Shortcut of symbol	Definition	Unit
$p_p$	Number of pole pairs	[-]
$R_a$	Phase resistance	[ $\Omega$ ]
$R_c$	Resistance represents core loss	[ $\Omega$ ]
$R_{eddy}$	Resistance represents eddy current loss	[ $\Omega$ ]
$R_h$	Resistance represents hysteresis loss	[ $\Omega$ ]
$R_{hb}$	Hysteresis resistance at base speed	[ $\Omega$ ]
$T_D$	Dynamic torque	[Nm]
$T_e$	Electromagnetic torque	[Nm]
$T_Z$	Load torque	[Nm]
$U_0$	Constraint voltage	[V]
$U_{0m}$	Maximal constraint voltage	[V]
$U_a$	Terminal voltage	[V]
$u_a, u_b, u_c$	Stator voltage at phase a, b, c respectively	[V]
$U_{am}$	Maximal terminal voltage	[V]
$u_d, u_q$	Voltage at d- and q-axis respectively	[V]
$u_{d0}, u_{q0}$	Voltage at shunt branch to resistance represents core loss in d-, q-axis	[V]
$u_{dc}, u_{qc}$	Voltage at resistance $R_c$ in d-, q-axis	[V]
$U_{nom}$	Rated rms line phase voltage	[V]
$\eta$	Efficiency	[-]
$\theta_0$	Initial angle between rotor direct axis and stator phase-a	[rad]
$\theta_e$	Angle between rotor direct axis and stator phase-a	[rad]
$v_{nom}$	Nominal winding temperature	[ $^{\circ}\text{C}$ ]
$\rho$	Saliency coefficient	[-]
$\tau_{FI}$	Time constant of frequency inverter	[s]
$\tau_q, \tau_d$	Time constant of PMSM in d or q axis respectively	[s]
$\tau_{SAMP}$	Time constant of frequency inverter sampler	[s]
$\tau_{SAMP2}$	Time constant of revolutions sampler	[s]
$\tau_{\sigma}, \tau_{\Sigma}$	Summary time constants	[s]
$\Psi_a$	Flux linkage due stator phase-a	[Wb]
$\psi_a$	Maximum flux linkage due to permanent magnet per phase	[Wb]
$\Psi_d, \Psi_q, \Psi_0$	Flux linkage due d-,q-axis and zero sequence respectively	[Wb]
$\Psi_{PM}$	Flux linkage due permanent magnets	[Wb]
$\omega_b$	Rotor electrical angular base speed	[rad.s <sup>-1</sup> ]
$\omega_e$	Rotor electrical angular speed	[rad.s <sup>-1</sup> ]
$\omega_m$	Rotor angular speed	[rad.s <sup>-1</sup> ]



# 1 INTRODUCTION

At last few years the Permanent Magnets Synchronous Machines (PMSM) became more attractive. There are many reasons for this fact. One of the biggest reasons for that is that price of rare-earth magnets is dropping. Also PMSMs efficiency is very high in compare with electric rotary machines without PM. [1]

Due to advantages, that PMSM has, it fit well for hybrid electric vehicles (HEV). [2] With massive developing in this area, requirements for drive control grow up. There are many ways to control drives with PMSM. Due this, there are some control strategies for different operation range. For example Zero D Axis Current controls method (CM), Maximum Torque Per Ampere CM, Maximum Efficiency CM, etc. These and other control methods will be discussed on in this thesis. [3]

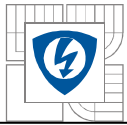
If some system has to be controlled, there must be description of this system. At this case, is necessary to describe PMSM mathematically. For this description is suitable to transform system from three phases abc reference frame to dq0 rotor reference frame. Reasons and advantages can be found at this thesis.

After transformation to dq0 system, equivalent circuit for each axis must be established. From these circuits can be mathematical equations derived and due this equations can be PMSM described. Most of equations are electrical, but there must be also used mechanical equations [4]-[7]. Due this, there are enough equations to solve the equation system. These are basics of model creation in Matlab-Simulink of PMSM.

Usually equivalent circuit in dq0 reference frame contains only resistance of stator winding, inductance in d and q axis respectively, cross couplings and back EMF from permanent magnets. For more accurately outcome, losses are in this thesis taken in consideration, due this are equivalent circuits little bit different and complicated against usual equivalent circuits of PMSMs [7]. In this thesis core losses are taken in consideration and are described.

There is a lot of ways, how to design controller for PMSM depending on area of operation. For every purpose, that PMSM will be used for is better to use different control strategy. In this thesis are used Optimal Module (OM) and Symmetric Optimum (SO) methods for design of angular speed and current controller.

After design of controllers, there is designed system extension which takes care about choosing most suitable control method for actual operation state.



## 2 PERMANENT MAGNET SYNCHRONOUS MACHINE

### 2.1 Advantages of PMSM

There are a lot of advantages of use permanent magnets instead of electric excitation system:

- No electrical energy is absorbed by the field excitation systems and thus there are no excitation losses, what means substantial increase in efficiency.
- Higher torque and output power per volume then when using electromagnetic excitation.
- Better dynamic performance than motor with electromagnetic excitation (higher magnetic flux density in the air gap).
- Simplification of construction and maintence.
- Reduction of prices.

Permanent magnets are improving the motor's steady-state performance and power density, dynamic performance and quality.

### 2.2 Construction

In dependence on how the rotor magnets are mounted onto (or inside) the rotor:

- Surface-mounted magnets
- Inset-mounted magnets
- Interior-mounted magnets
- Rotor with buried magnets symmetrically distributed
- Rotor with buried magnets asymmetrically distributed
- Bread loaf magnets
- Decentered magnets
- Interior double-layer magnets

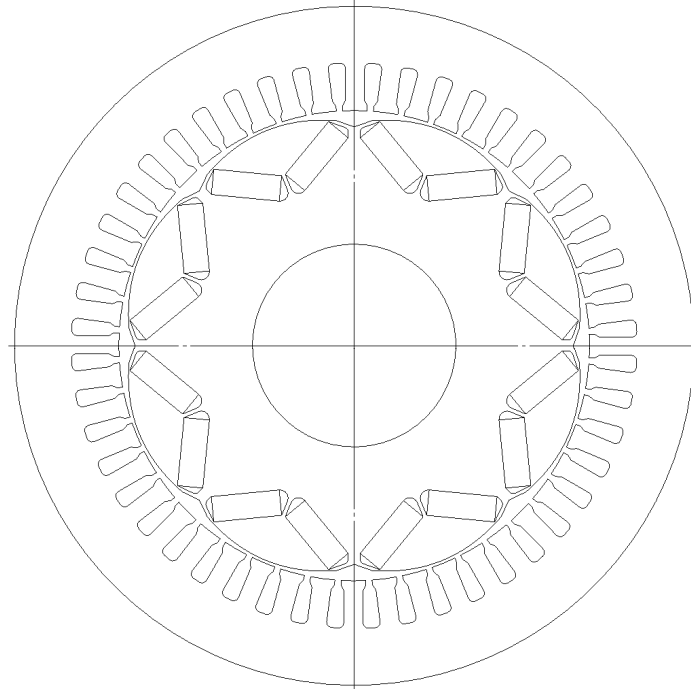
In dependence on which magnets are used:

- Alnico magnets (Al, Ni, Co, Fe)
- Ferrite magnets
- Rare-earth materials (SmCo, NdFeB)
- Etc...

[1] [6]

In this thesis PMSM with embedded magnets placed in V position as is shown on Fig. 2.1 is used. At this rotor form, two permanent magnets magnetize the same pole. The pole shoes are shaped to produce sine-wave air-gap flux density. Advantages of rotor with magnets placed into V position are that machine produces more torque per rotor volume due to high air-gap flux density, compared to the rotor with surface mounted magnets. And there are other advantages like there isn't danger of magnets coming off. There is no problem with fixing the magnets to the

rotor as is at surface rotor machine. The biggest advantage of this rotor is that air-gap flux density can be easily made sinusoidal which makes it possibilities to achieve a very low cogging torque. [8]



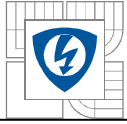
*Fig. 2.1: Cross section of used PMSM*

In table 2.1 are parameters of equivalent circuit of used PMSM:

$U_{nom}[V]$	102
$I_{nom}[A]$	203,7
$p_p[-]$	4
$R_a[\Omega]$	0,0281
$\Psi_{PM}[Wb]$	0,1883
$L_d[mH]$	0,3286
$L_q[mH]$	0,6089
$R_{hb}[\Omega]$	95,73
$R_{eddy}[\Omega]$	82,21
$J[kgm^2]$	0,147
$v_{nom}[^{\circ}C]$	160

*Table 2.1: Parameters of equivalent circuit of PMSM*





## 3 TRANSFORMATION OF THREE PHASES SYSTEM

### 3.1 Advantages of Park's transformation

Main reason, why to transform three phases a,b,c system to dq0 is to simplify description of system. Instead of equation for each phase, there will be only, two dimensional system with direct- and quadrature-axis.

Main advantage of using dq0 system is easy analysis of the interaction of rotor and stator flux and mmf waves, independent of whether or not saliency effects are present. By transforming a,b,c quantities into equivalent quantities which rotate in synchronism with the rotor steady state conditions, these interaction become those of constant mmf- and flux-waves separated by a constant spatial angle. This indeed is a point of view, which correspond to that of an observer in the rotor reference frame.

Park transformation is used to transform three phase time variable quantities (in this case for example  $u_a, u_b, u_c$ ) to direct-quadrature-axis and zero sequence component (in this case  $u_d, u_q$  and  $u_0$ ). The zero sequence component is important if three phase system is not balanced. If it is, the zero sequence components are neutral. In this thesis is only balanced three phase system take in consideration, so 0 sequence components are not discussed in any detail. [9]

### 3.2 Park's transformation

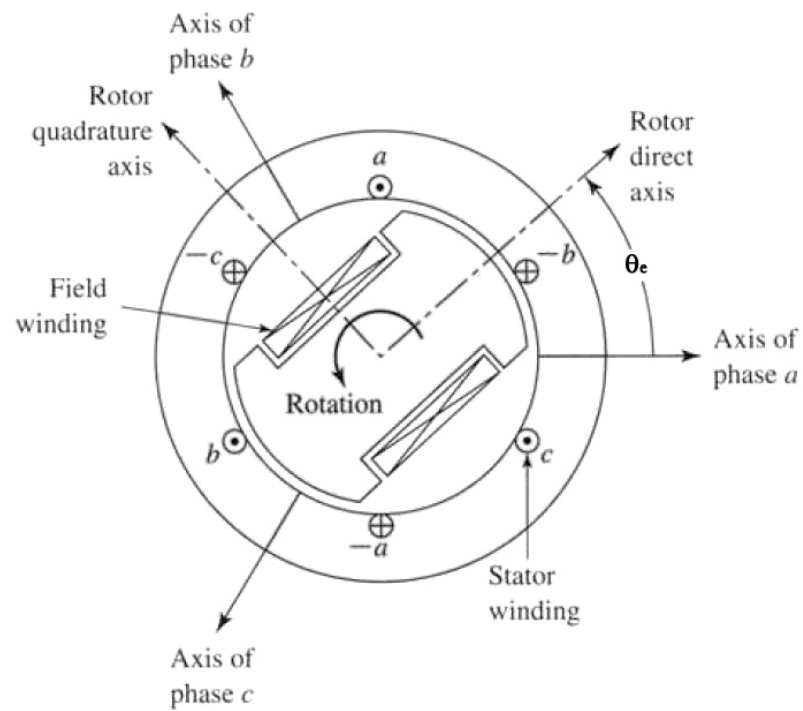
For transformation of any stator quantity (voltage, current, flux linkage, etc.) the transformation matrix is established.

$$\begin{bmatrix} X_d \\ X_q \\ X_0 \end{bmatrix} = \begin{bmatrix} k_d \cos(\theta_e) & k_d \cos(\theta_e - 120^\circ) & k_d \cos(\theta_e + 120^\circ) \\ -k_q \sin(\theta_e) & -k_q \sin(\theta_e - 120^\circ) & -k_q \sin(\theta_e + 120^\circ) \\ k_0 & k_0 & k_0 \end{bmatrix} \cdot \begin{bmatrix} X_a \\ X_b \\ X_c \end{bmatrix} \quad (3.1)$$

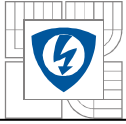
And the inverse transformation as

$$\begin{bmatrix} X_a \\ X_b \\ X_c \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \frac{1}{k_d} \cos(\theta_e) & -\frac{2}{3} \frac{1}{k_q} \sin(\theta_e) & \frac{1}{3k_0} \\ \frac{2}{3} \frac{1}{k_d} \cos(\theta_e - 120^\circ) & -\frac{2}{3} \frac{1}{k_q} \sin(\theta_e - 120^\circ) & \frac{1}{3k_0} \\ \frac{2}{3} \frac{1}{k_d} \cos(\theta_e + 120^\circ) & -\frac{2}{3} \frac{1}{k_q} \sin(\theta_e + 120^\circ) & \frac{1}{3k_0} \end{bmatrix} \cdot \begin{bmatrix} X_d \\ X_q \\ X_0 \end{bmatrix} \quad (3.2)$$

At equation (3.2) X represent any stator quantity to be transformed. Constants  $k_d, k_q$  and  $k_0$  are transformation constants and they can be chosen absolutely different from each other, but there is no reason to choose them different. There are advantages to choose  $k_d=k_q=2/3$  and  $k_0=1/3$  which will be show in next section. These values was used by Park because of simplification of equations (3.2). Fig. 3.1 shows simplified sketch of synchronous machine with wound rotor with one pole pair for simpler understand Park's transformation.



*Fig. 3.1: Simplified cross section of synchronous machine with wound rotor [9]*



## 4 DESCRIPTION OF PMSM

### 4.1 Transformation stator voltages to dq0 rotor reference frame

Equations for stator voltages of PMSM are:

$$u_k = R_k i_k + \frac{d\psi_k}{dt} \quad (4.1)$$

where

$$k = a, b, c \quad (4.2)$$

The transformation of stator voltages to d-q voltages is proceeding from equation for flux linkage due to phase A

$$\psi_a = \frac{2}{3} \frac{1}{k_d} \psi_d \cos(\theta_e) - \frac{2}{3} \frac{1}{k_q} \psi_q \sin(\theta_e) + \frac{1}{3} \frac{1}{k_0} \psi_0 \quad (4.3)$$

Now let's do the differentiation of equation (4.3) with respect to time when:

$$\theta_e = \omega_e t + \theta_0, \theta_0 = 0 \quad (4.4)$$

Equation (4.3) will go to:

$$\frac{d\psi_a}{dt} = \frac{2}{3} \frac{1}{k_d} \frac{d\psi_d}{dt} \cos(\theta_e) - \frac{2}{3} \frac{1}{k_d} \psi_d \omega_e \sin(\theta_e) - \frac{2}{3} \frac{1}{k_q} \frac{d\psi_q}{dt} \sin(\theta_e) - \frac{2}{3} \frac{1}{k_q} \psi_q \omega_e \cos(\theta_e) + \frac{1}{3k_0} \frac{d\psi_0}{dt} \quad (4.5)$$

From equation (4.1) is evidently that:

$$\frac{d\psi_a}{dt} = u_a - R_a i_a \quad (4.6)$$

So when substitute  $u_a$  and  $i_a$  from definition of Park's transformation into (4.6) it will be:

$$\frac{d\psi_a}{dt} = \frac{2}{3} \frac{1}{k_d} u_d \cos(\theta_e) - \frac{2}{3} \frac{1}{k_q} u_q \sin(\theta_e) + \frac{1}{3k_0} u_0 - R_a \left( \frac{2}{3} \frac{1}{k_d} i_d \cos(\theta_e) - \frac{2}{3} \frac{1}{k_q} i_q \sin(\theta_e) + \frac{1}{3k_0} i_0 \right) \quad (4.7)$$

Equation (4.5) is equal to (4.7) if coefficients with the same trigonometric function are the same. So if these coefficients are compared under condition that  $k_d = k_q = 2/3$  and  $k_0 = 1/3$  it's possible to find equations for voltages at d and q axis.

$$\begin{aligned} \frac{d\psi_d}{dt} - \psi_q \omega_e &= u_d - R_a i_d \\ u_d &= R_a i_d + \frac{d\psi_d}{dt} - \omega_e \psi_q \end{aligned} \quad (4.8)$$



Where

$$\begin{aligned}\psi_d &= L_d i_d + \psi_{PM} \\ \psi_q &= L_q i_q\end{aligned}\quad (4.9)$$

Institution (4.9) into (4.8) leads to

$$\underline{u_d = R_a i_d + L_d \frac{di_d}{dt} - \omega_e L_q i_q} \quad (4.10)$$

$$-\omega_e \psi_d - \frac{d\psi_q}{dt} = R_a i_q - u_q \quad (4.11)$$

$$u_q = R_a i_q + \frac{d\psi_q}{dt} + \omega_e \psi_d$$

Institution (4.9) into (4.11) leads to

$$\underline{u_q = R_a i_q + L_q \frac{di_q}{dt} + \omega_e L_d i_d + \omega_e \psi_{PM}} \quad (4.12)$$

$$\begin{aligned}\frac{d\psi_0}{dt} &= u_0 - R_a i_0 \\ \underline{u_0} &= R_a i_0 + \frac{d\psi_0}{dt}\end{aligned}\quad (4.13)$$

Equations(4.10), (4.12) and (4.13) describe synchronous machine mathematically without take in consideration core losses. [10]. Due these facts it is possible to draw equivalent circuit for PMSM in d- and q-axis.

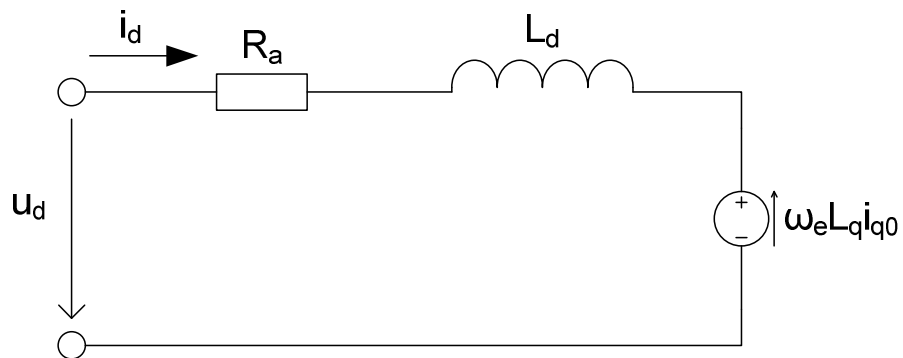


Fig. 4.1: Equivalent circuit of PMSM at d-axis

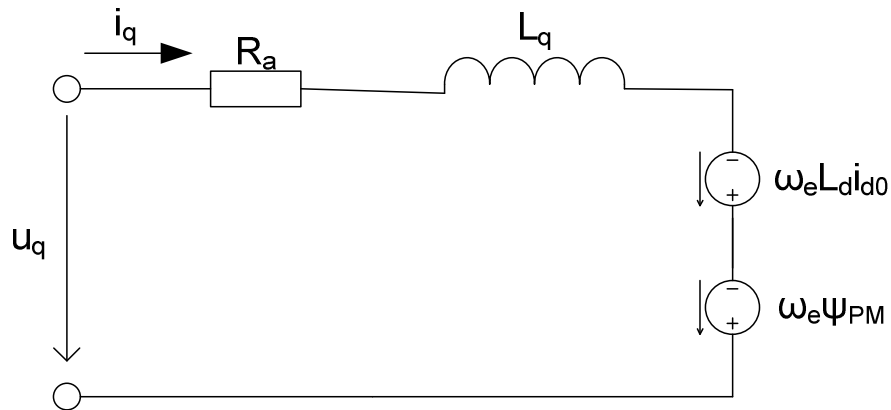


Fig. 4.2: Equivalent circuit of PMSM at q-axis

## 4.2 Losses at PMSM

In previous section were derived equivalent circuits for PMSM in dq0 axis. At these circuits aren't taken in consideration core losses. But concretely at electric or hybrid cars is efficiency very important. That's the reason, why losses must be taken in consideration. In this section will be discussed losses, which are important to take in consideration.

The most dominant are copper and core losses in PMSM. The main sources of core losses are eddy currents and hysteresis losses.

### 4.2.1 Copper losses

Copper losses result from Joule heating and so are also referred to as the stator resistance point square of stator current. With high-frequency currents, winding loss is affected by proximity effect and skin effect, and cannot be calculated as simply.

### 4.2.2 Eddy currents losses

Eddy currents are caused by induction of current inside the stator. They are nearly proportional to the square of the product of air gap flux linkages and frequency of the flux variation. For this thesis are eddy current losses represent by resistance  $R_{\text{eddy}}$  which is constant for any rotor speed.

### 4.2.3 Hysteresis losses

Hysteresis losses are the result of continuous variation of flux linkages in the core. They are nearly proportional to the product of square of flux linkages and frequency of flux variation. In this thesis hysteresis are losses represented by resistance  $R_h$ , which is defined by equation

$$R_h = R_{h,b} \frac{\omega_e}{\omega_b} \quad (4.14)$$

where  $R_{h,b}$  is resistance represents hysteresis losses at base speed of 1300rpm,  $\omega_e$  is rotor electrical angular speed and  $\omega_b$  is base rotor electrical angular speed.

#### 4.2.4 Core losses

Core losses are mainly compound from eddy currents and hysteresis losses. Core losses can be represented as parallel combination of resistances represents eddy current losses and hysteresis losses. Core losses are obviously negligible at very low speeds what imply from its dependence on hysteresis losses and its dependence on rotor electric angular speed.

$$R_c = \frac{R_{eddy} \cdot R_h}{R_{eddy} + R_h} \quad (4.15)$$

#### 4.2.5 Stray losses

Stray losses are the result of distortion of the air gap flux by the phase current. Non-uniform distribution of current in the copper also leads to stray losses. Stray losses are very difficult to estimate. Therefore, these losses are usually bundled with core losses during modeling or during experimental measurements. In this thesis won't be taken in consideration.

### 4.3 Equivalent circuits of PMSM with losses

If losses that have been mentioned in 4.2 will be taken in consideration, it is necessary to transform equivalent d and q circuits to circuit, where these losses will be mentioned:

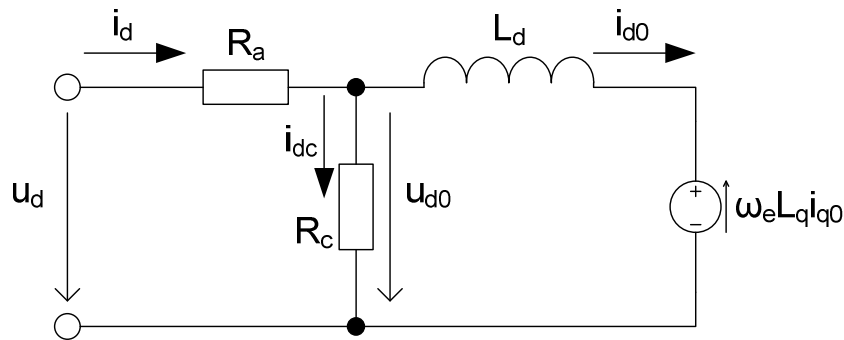


Fig. 4.3: Equivalent d axis circuit of PMSM with losses

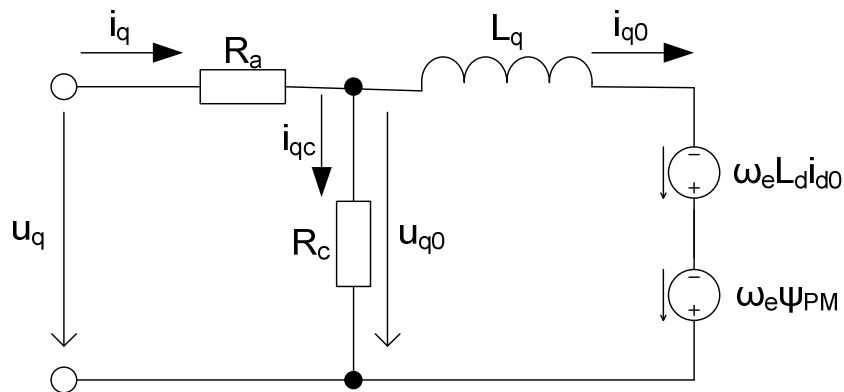
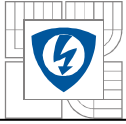


Fig. 4.4: Equivalent q axis circuit of PMSM with losses



Equations describing PMSM with losses can be derived from circuits for d and q axes shown on Fig. 4.3 and Fig. 4.4. Each circuit is described by 2 equations:

$$-u_d + R_a i_d + R_c (i_d - i_{d0}) = 0 \quad (4.16)$$

$$R_c (i_{d0} - i_d) + L_d \frac{di_{d0}}{dt} - \omega_e L_q i_{q0} = 0 \quad (4.17)$$

Equations (4.16) and (4.17) describe d axis of PMSM. Their solution leads to only one equation, which simplifies description of d axis (equation (4.19)):

$$i_d = \frac{u_d + i_{d0} R_c}{R_a + R_c} \quad (4.18)$$

$$\frac{di_{d0}}{dt} = \frac{1}{L_d} \left( R_c \frac{u_d + R_c i_{d0}}{R_a + R_c} - R_c i_{d0} + \omega_e L_q i_{q0} \right) \quad (4.19)$$

The same procedure as was used to describe d axis circuit can be used for q axis circuit, but of course with parameters of q axis:

$$-u_q + R_a i_q + R_c (i_q - i_{q0}) = 0 \quad (4.20)$$

$$R_c (i_{q0} - i_q) + L_q \frac{di_{q0}}{dt} + \omega_e (\Psi_{PM} + L_d i_{d0}) = 0 \quad (4.21)$$

$$i_q = \frac{u_q + R_c i_{q0}}{R_a + R_c} \quad (4.22)$$

$$\frac{di_{q0}}{dt} = \frac{1}{L_q} \left[ R_c \frac{u_q + R_c i_{q0}}{R_a + R_c} - R_c i_{q0} - \omega_e (\Psi_{PM} + L_d i_{d0}) \right] \quad (4.23)$$

Equation (4.23) is solution, which describe q axes of PMSM. From Fig. 4.3 and Fig. 4.4 also passes following equations given by Kirchhoff laws:

$$i_{d0} = i_d - i_{dc} \quad (4.24)$$

$$i_{q0} = i_q - i_{qc} \quad (4.25)$$

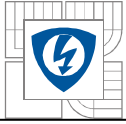
Where currents  $i_{dc}$  and  $i_{qc}$  can be represented as:

$$i_{dc} = -\frac{\omega_e \rho L_d i_{q0}}{R_c} \quad (4.26)$$

and

$$i_{qc} = \frac{\omega_e (\Psi_{PM} + L_d i_{d0})}{R_c} \quad (4.27)$$

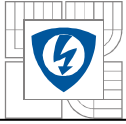
where  $\rho = L_q / L_d$ .



$$T_e = \frac{3}{2} p_p \left[ \Psi_{PM} i_{q0} + (L_d - L_q) i_{d0} i_{q0} \right] \quad (4.28)$$

Equation (4.28) completes description of PMSM in dq0 reference frame as a mechanical equation adjusted for model with losses.





## 5 CONTROL STRATEGIES

### 5.1 Zero D axis current control strategy

Zero D axis control strategy is the most widely utilized control strategy by the industry. In this strategy is D axis current maintained at zero. Due this, the torque control mechanism is simplified, because of linearizing the relationship between torque and current. This is the main advantage of this control strategy. That mean, that linear torque controller can be used. All these facts mean that this control strategy of PMSM is very similar to control of DC motors.

If d axis current is equal zero, electromagnetic torque will be:

$$T_e = \frac{3}{2} p_p \cdot i_{q0} \psi_{PM} \quad (5.1)$$

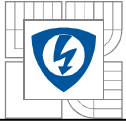
That means, that torque can be driven by  $i_{q0}$ , and that's big advantage, because torque become linear depending on current  $i_{q0}$ .

### 5.2 Maximum torque per ampere control strategy (MTPA)

The MTPA control strategy is the most widely studied control strategy by the research community. For given torque, this method minimizes current. Therefore, copper losses are minimized in the process. The maximum possible torque under this control strategy is only limited by the maximum possible power loss.

### 5.3 Maximum Efficiency Control Strategy (ME)

In this method, d and q axis currents are coordinated to minimum core power loss at any operating speed. This could be done using numerical methods. On the other hand, minimizing copper losses is equivalent to minimizing current. Therefore, the maximum efficiency control strategy results in minimum current for a given torque at zero speed, which means that the ME and MTPA control strategies result in identical performance at zero speed. [3]



## 6 CONTROLLER DESIGN FOR NO LOSSES PMSM

This chapter deals with design current and revolutions controller for no losses model of PMSM. For design controller of PMSM is necessary to compute transfer function of PMSM. For getting this function, there is to be used equivalent circuit with neglected core losses. Main reason for that, is simplification of controllers design. There are only copper losses taken in consideration. Due to this, equivalent circuit comes simplified like shows Fig. 4.1 and Fig. 4.2. From these circuits can be derived equations describing PMSM. From equations (4.10) and (4.11) can be easily derived equations for currents in each axis:

$$\frac{di_d}{dt} = \frac{1}{L_d} (u_d - R_a i_d + \omega_e L_q i_q) \quad (6.1)$$

for D axis and

$$\frac{di_q}{dt} = \frac{1}{L_q} (u_q - R_a i_q - \omega_e L_d i_d - \omega_e \psi_{PM}) \quad (6.2)$$

for Q axis.

Last equation, which describes PMSM, is mechanical equation:

$$T_e = T_D + T_Z \quad (6.3)$$

where

$$T_D = \frac{J}{p} \frac{d\omega_e}{dt} \quad (6.4)$$

and

$$T_e = \frac{3}{2} p_p \left[ i_q \psi_{PM} + (L_d - L_q) i_d i_q \right] \quad (6.5)$$

from equations (6.3), (6.4) and (6.5) is given equation for electric angular speed of PMSM:

$$\frac{d\omega_e}{dt} = \frac{1}{J} p (T_e - T_Z) \quad (6.6)$$

Due to equations (6.1), (6.2) and (6.6) can be build up model of PMSM (Fig. 6.1).

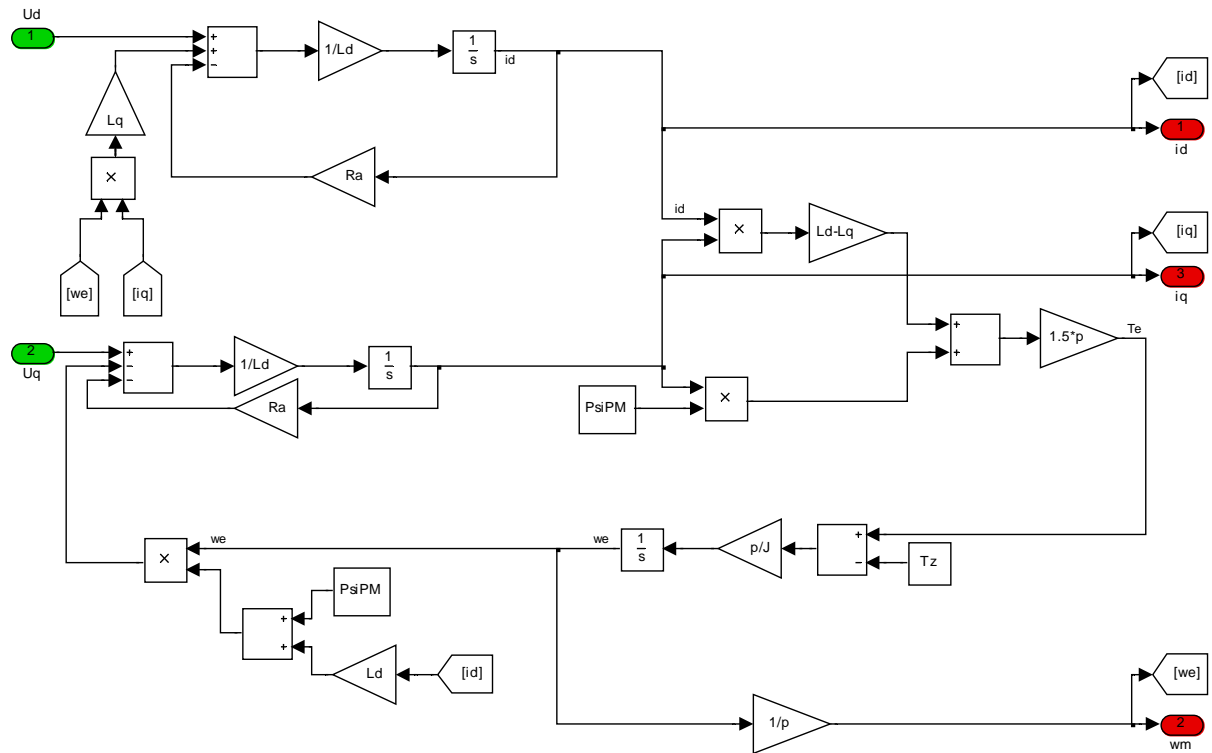


Fig. 6.1: PMSM model without losses

After setting up of PMSM model, next step is to create transfer function of PMSM, because of design of controllers. Due the reason, that design of controller is based on zero d axis current strategy, let's focus to q axis. q axis is described by equation (4.12). Because of premise that  $i_d=0$  and  $\omega=0$ , equation (4.12) will go to:

$$u_q = R_a i_q + L_q \frac{di_q}{dt} \quad (6.7)$$

After Laplace transformation, will be equation (6.7) modified to the form:

$$u_q = R_a i_q + p L_q i_q \quad (6.8)$$

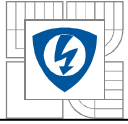
So transfer function for q axis is:

$$\frac{i_q}{u_q} = \frac{1}{R_a + p L_q}; \tau_q = \frac{L_q}{R_a} \quad (6.9)$$

For d axis is procedure the same, but instead of  $L_q$  is  $L_d$ :

$$\frac{i_d}{u_d} = \frac{1}{R_a + p L_d}; \tau_d = \frac{L_d}{R_a} \quad (6.10)$$

For complete description of system, is important to establish transfer function of frequency converter and sampler. These transfer function are established in appendix C. If all transfer



functions are known, it is possible to establish complete transfer function of full system which is shown on Fig. 6.2.

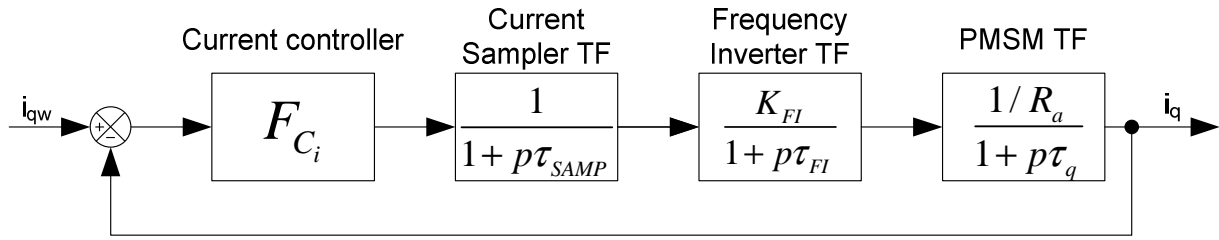


Fig. 6.2: Transfer function of full system with current controller

Due Fig. 6.2 can be written equation of transfer function in open loop of system without current controller:

$$F_S = \frac{K_{FI} \cdot \frac{1}{R_a}}{(1 + p\tau_{SAMP})(1 + p\tau_{FI})(1 + p\tau_q)} \quad (6.11)$$

Let's establish time constant  $\tau_\sigma$ :

$$\tau_\sigma = \tau_{SAMP} \quad (6.12)$$

Due this, equation (6.11) will be simplified:

$$F_S = \frac{K_{FI} \cdot \frac{1}{R_a}}{(1 + p\tau_\sigma)(1 + p\tau_{FI})(1 + p\tau_q)} \quad (6.13)$$

This is final transfer function of PMSM including frequency inverter.

## 6.1 Current controller design

To design current controller can be used two methods. First is called Symmetric Optimum (SO) and second is called Optimal Module (OM). For current controller is better to use second one, because it is slower, but there is only 4,3% overshoot of regulated quantity. And that is important. First one is faster, but overshoot is much bigger (about 43,3%).

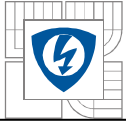
Transfer function of solution due the OM method is:

$$F_{OM} = \frac{1}{2p\tau_\sigma(1 + p\tau_\sigma)} \quad (6.14)$$

Transfer function of system with controller is:

$$F_{OM} = F_{Ci} \cdot F_S \quad (6.15)$$

Now it is possible to declare  $F_{Ci}$  from (6.15) and institute there from (6.14) and (6.13):



$$F_{C_i} = \frac{1}{F_s} F_{OM} = \frac{(1 + p\tau_q)(1 + p\tau_{FI})}{2\tau_\sigma \frac{1}{R_a} K_{FI} p} = \underbrace{\frac{\tau_q + \tau_{FI}}{\tau_0}}_p + \underbrace{\frac{1}{\tau_0}}_I \frac{1}{p} + \underbrace{\frac{\tau_q \cdot \tau_{FI}}{\tau_0}}_D p \quad (6.16)$$

## 6.2 Revolutions controller design

To design revolutions controller will be used first method – Symmetric Optimum, because in it is faster, and in case of revolutions is overshoot doesn't matter so much. On Fig. 6.3 is shown full system scheme of revolutions loop.

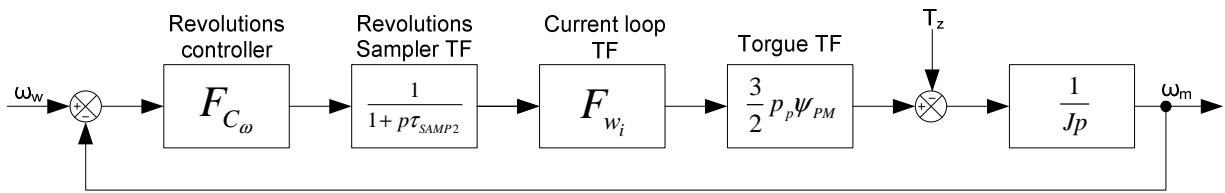


Fig. 6.3: Revolutions controller loop

Design procedure is the same as in case of Optimal Module method, only transfer function of open loop in case of Symmetric Optimum is different:

$$F_{SO} = \frac{1 + 4p\tau_\Sigma}{8p^2\tau_\Sigma^2(1 + \tau_\Sigma)} \quad (6.17)$$

Transfer function of open loop of system without controller:

$$F_s = \frac{\frac{1}{Jp} \cdot \frac{3}{2} \psi_{PM} p_p}{(1 + p\tau_{SAMP2})(1 + 2p\tau_\sigma)} \quad (6.18)$$

Let's replace time constants with one  $\tau_\Sigma$ :

$$\tau_\Sigma = 2\tau_\sigma \quad (6.19)$$

Revolutions controller due SO method is designed as:

$$F_{C_\omega} = \frac{1}{F_s} F_{SO} = \frac{(1 + p\tau_{SAMP2})(1 + 4p\tau_\Sigma)}{\frac{1}{J} 12 \cdot \Psi_{PM} \cdot p_p \cdot 4 \cdot \tau_\Sigma^2 \cdot p} = \underbrace{\frac{4\tau_\Sigma + \tau_{SAMP2}}{\tau_1}}_p + \underbrace{\frac{1}{\tau_1}}_I \frac{1}{p} + \underbrace{\frac{4p\tau_\Sigma \cdot \tau_{SAMP2}}{\tau_1}}_D p \quad (6.20)$$

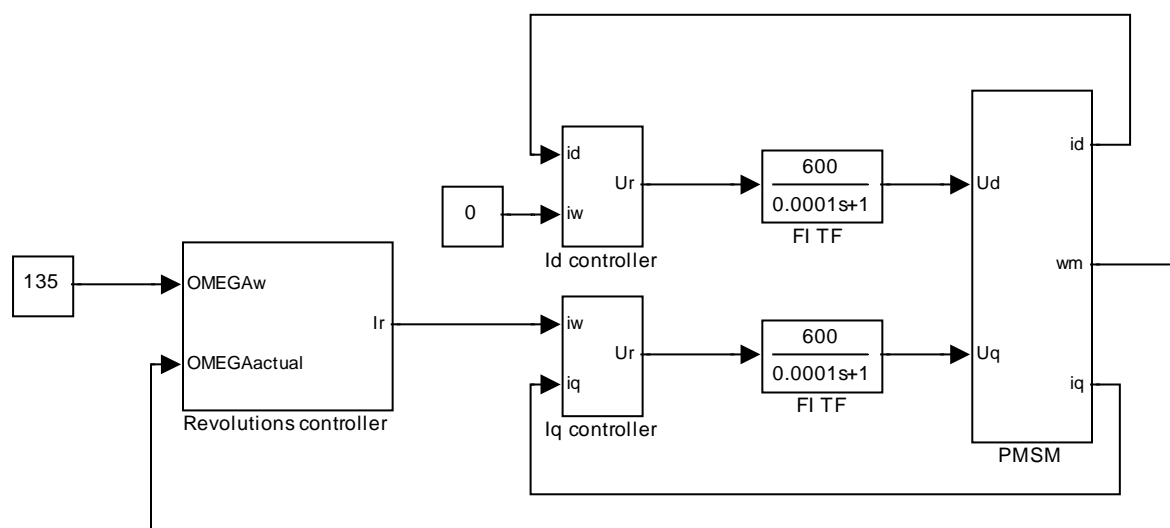
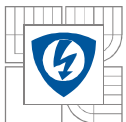


Fig. 6.4: PMSM with revolutions and current controllers ( $i_d=0$  control strategy)



## 7 CONTROLLER DESIGN FOR PMSM WITH LOSSES

Equations (4.14), (4.15), (4.19), (4.23), (4.28) and (6.6) were used for establish model of PMSM with losses. For getting transfer function of this system will be used same method and conditions, as for non losses system, but with different system parameters. Transfer function for q axis will be derived from (4.23) which will be after simplification:

$$u_q = i_{q0} \cdot p \frac{L_q}{R_c} + i_{q0} \cdot R_a \quad (7.1)$$

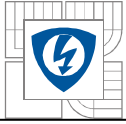
So transfer function will be:

$$F_{SL} = \frac{i_{q0}}{u_q} = \frac{1}{R_a} \frac{1}{1 + p \frac{L_q}{\underbrace{R_a \cdot R_c}_{\tau_{qL}}}} = \frac{\frac{1}{R_a}}{1 + p \tau_{qL}} \quad (7.2)$$

For d axis is transfer function almost the same, but in time constant is in numerator  $L_d$ . As you can see, transfer function of PMSM with losses is almost the same as for non-losses model. The only difference is in the way of calculation of time constants  $\tau_q$  and  $\tau_d$ .<sup>1</sup>

---

<sup>1</sup> For purposes of better results were some time constants changed. how was they changed is clear from configuration M-file which is placed in Appendix J. Also D components of controllers were neglected for its small values.



## 8 CONTROL MODE SWITCHING BLOCK DESIGN

### 8.1 Main purpose of switching block

In this chapter will be described switching block which is evaluate variables in time and accordance with these variables is switching control method between MTPA and Flux Weakening method.

If drive should be driven by MTPA control method, there must be realized few conditions. MTPA control method is suitable for two kinds of operation states. First one is when drive works with revolutions lower then basic revolutions. Another operating area is, when wanted revolutions are higher than base revolutions of the machine. In this case is important, if frequency inverter is still able to afford sufficient voltage. If yes, it will be used MTPA control strategy.

Flux weakening control strategy will be chosen if revolutions of rotor are higher then over excitation threshold speed  $\omega_c$  or if needed voltage is higher than voltage that is available from inverter.

### 8.2 Design switching block

The armature current is given as:

$$I_a = \sqrt{i_d^2 + i_q^2} \quad (8.1)$$

and it is always lower or equal to maximal current given by inverter  $I_{am}$ . Same situation is with terminal voltage:

$$U_a = \sqrt{u_d^2 + u_q^2} \quad (8.2)$$

and it also is always lower or equal to maximum available terminal voltage from inverter depending on DC link voltage.

As the PMSM has saliency and, as a result, the reluctance torque is available the armature current vector is controlled in order to produce the maximum torque per armature ampere [12]. From (4.10) and (4.11) the relationship between  $i_d$  and  $i_q$  for MTPA control is derived as:

$$i_d = \frac{\psi_a}{2(Lq - Ld)} - \sqrt{\frac{\psi_a^2}{4(Lq - Ld)^2} + i_q^2} \quad (8.3)$$

where  $\Psi_a = \frac{3}{2} \Psi_{PM}$ . The maximum torque per ampere is produced when  $I_a = \sqrt{i_d^2 + i_q^2} = I_{am}$ .



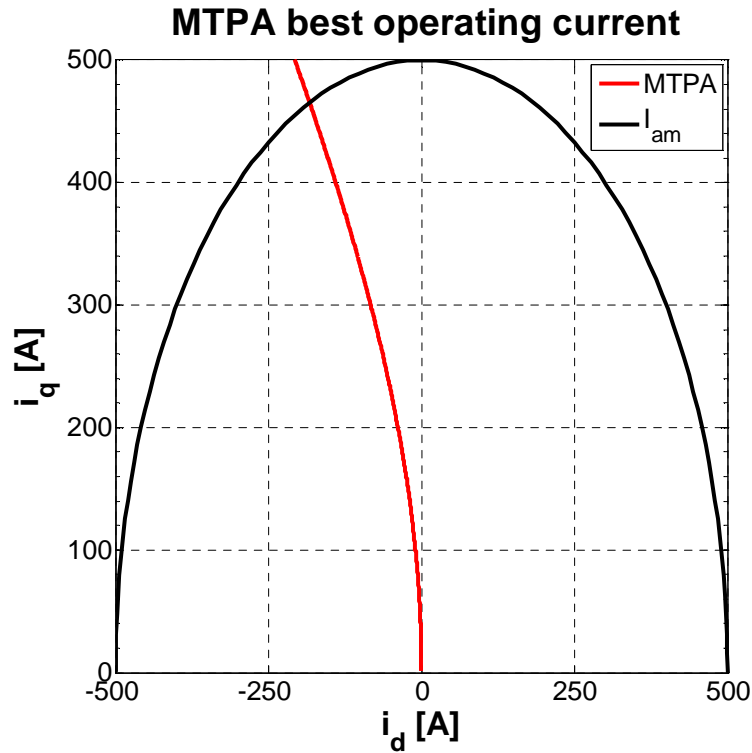


Fig. 8.1: Ideal ratio of  $i_d$  and  $i_q$  for MTPA

This situation is shown on Fig. 8.1. Maximum torque will be at moment of crossing trajectory MTPA and current limit circuit which was for example chosen 500A. Because of simplification will be resistance of armature neglected and voltage constrain will be for now defined as:

$$U_0 = \sqrt{u_{d0}^2 + u_{q0}^2} \leq U_{om} \quad (8.4)$$

where

$$u_{do} = -\omega_e L_q i_q, u_{qo} = \omega_e L_d i_d + \omega_e \Psi_a \quad (8.5)$$

and

$$U_{om} = U_{am} - R_a I_{am} \quad (8.6)$$

The d and q axis components of armature are controlled in order to keep  $U_o$  equal to  $U_{om}$  in flux weakening constant power region. Then from (8.4) and from condition of  $U_o = U_{om}$  can be derivate relationship between  $i_d$  and  $i_q$ :

$$i_d = -\frac{\Psi_a}{L_d} + \frac{1}{L_d} \sqrt{\frac{U_{om}^2}{\omega_e^2} - (L_q i_q)^2} \quad (8.7)$$

If the current vector is controlled according to (8.7), the resultant terminal voltage  $U_a$  is always kept within  $U_{am}$  in steady state. So in dependence on in which area of operation is machine, switching block recognize which method is for given area optimal. Flow chart of decision process is shown on Fig. 8.2.

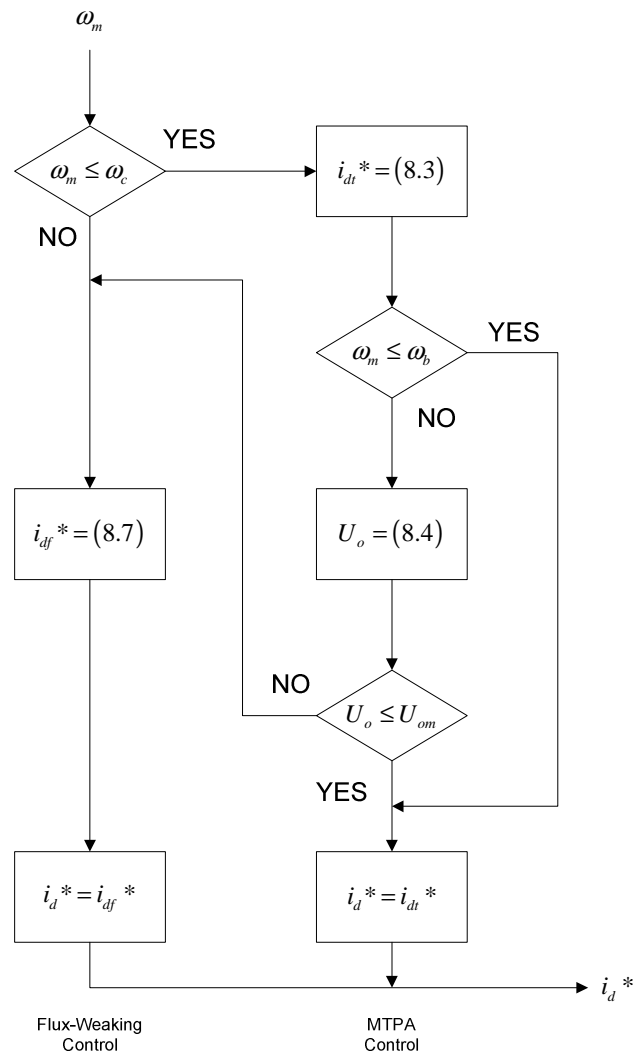


Fig. 8.2: Flow chart of switching block

Information about how is this bloc realized for purposes of control can be found in Appendix C.



## 9 EFFICIENCY CALCULATION

For calculation of efficiency will be used armature current  $I_a$  as is defined in (8.1) and terminal voltage  $U_a$  defined as:

$$U_a = \sqrt{\left(R_a i_d - \omega_e \rho L_d i_{0q}\right)^2 + \left(R_a i_q + \omega_e (L_d i_{d0} + \Psi_{PM})\right)^2} \quad (9.1)$$

where  $\rho = L_q / L_d$  is a saliency coefficient. Then it is possible to write equation for torque as:

$$T_e = p_p \cdot \Psi_a \cdot i_{0q} + p_p (1 - \rho) L_d i_{0d} i_{0q} \quad (9.2)$$

First term in(9.2) represents the magnetic torque and second term is the reluctance torque.

Due to Fig. 4.3, Fig. 4.4 and equations (4.24), (4.25), (4.26) and (4.27) can be established copper losses  $P_{Cu}$ , the iron losses  $P_{Fe}$ , and mechanical losses  $P_M$ :

$$P_{Cu} = R_a (i_d^2 + i_q^2) = R_a \left[ \left( i_{d0} - \frac{\omega \rho L_d i_{q0}}{R_c} \right)^2 + \left( i_{q0} + \frac{\omega_e (\Psi_{PM} + L_d i_{d0})}{R_c} \right)^2 \right] \quad (9.3)$$

$$P_{Fe} = R_c (i_{dc}^2 + i_{qc}^2) = \frac{\omega_e^2 (\rho L_d i_{q0})^2}{R_c} + \frac{\omega_e^2 (\Psi_{PM} + L_d i_{d0})^2}{R_c} \quad (9.4)$$

$$P_M = T_M \cdot \omega_m \quad (9.5)$$

Then electrical losses are given as a sum of iron and copper losses

$$P_E = P_{Cu} + P_{Fe} \quad (9.6)$$

And total losses are:

$$P_L = P_E + P_M \quad (9.7)$$

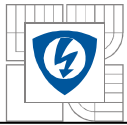
The output power is expressed as:

$$P = T_e \cdot \omega_r \quad (9.8)$$

Finally efficiency is will be expressed as:

$$\eta = \frac{P}{P + P_L} \cdot 100\% \quad (9.9)$$

Subsystem that calculate efficiency in Matlab Simulink is shown in Appendix D.



## 10 SIMULATIONS

On established models of Permanent Magnet Synchronous Machines is possible simulate almost every state of operation. For example there are chosen 3 simply situations which are simulated. Every state was simulated with all three control strategies because of possibility to compare behavior between each other. Only last one wasn't simulated on PMSM without losses.

### 10.1 Start up of PMSM

In this case is simulated start up of PMSM with constant braking torque of value 200Nm. In Appendix F are shown results for this simulation.

### 10.2 Start up of PMSM and elevation of braking torque

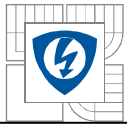
This simulation begins in the same way as first one, but after stabilizing revolutions and all another quantities, motor's load is raised from 100Nm to 200Nm. Results are also shown in Appendix G.

### 10.3 Switch from MTPA to FW

At this case, there is used ramp as a input for wanted angular speed. Until angular speed is under set value, PMSM is driven by MTPA, then in first sector of operation, switching block decide which of control strategies is better to use. After crossing second sector, it will switch definitely to FW control strategy. Results are also shown in Appendix H.

### 10.4 Efficiency comparing

This set of simulations is focused to explore influence of efficiency of PMSM in dependence on braking load and control strategy. Results can be found in conclusion.



## 11 CONCLUSION

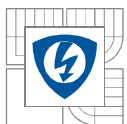
In this thesis were basic kinds of permanent magnets synchronous machine introduced and advantages of usage of permanent magnets at rotary electric machines were described. As was shown, for design model of PMSM is advantageous transformation from abc to dq0 rotor reference frame, because of simplify mathematical description. Concretely Park's transformation was used to transform reference frame and was showed how to establish equations of PMSM in dq0 reference frame due to Parks transformation.

These derived equations were modified, to the form which describes model of PMSM with losses takes in consideration. These electrical and mechanical equations of PMSM had to be established, because of completely description of PMSM. Due these equations, Matlab simulink model could be build. Model of PMSM and PMSM with losses is shown in Appendix A and B respectively.

In next part of this thesis was described drive system consist of PMSM, frequency inverter and revolutions sensor. For this system were derived transfer functions and due this was possible to design current control for d and q axis. Then was system described again and designed control of rotor angular speed. This system was modeled in Matlab-Simulink and simulations on these systems were made. Concretely were tested properties of different control strategies. From results it stands for reason, that  $i_d=0$  control strategy is most simply one, but in compare to MTPA control strategy the has worst efficiency. Also FW control strategy was tested in combination with MTPA control strategy. From results which are shown in Appendix H is evidently, that it is useful way of control salient PMSM in case of high revolutions or when frequency inverter can't afford so high energy which is necessary for wanted rotations.

From results of simulations where efficiency was calculated is evidently, that efficiency was around 95 or 96%. These values are higher than real efficiencies of PMSM motors. One of reasons for that is that there were neglected some negative factors that influencing efficiency. For example there weren't taken in consideration mechanical losses. But in the end, it wasn't have any influence for comparing control strategies between each other, because all simulations was made on same model. That means, comparing control strategies with each other can be taken as serious and it is possible to take these results as useful.

From graph that is placed in Appendix I is evidently advantage of MTPA control strategy against  $i_d=0$ . It is clear, that efficiency with growing braking load is higher when MTPA control strategy is used then  $i_d=0$ . There is also shown efficiency of FW control strategy, but it can't be compared to previous two control methods because of different condition of simulation when  $\omega_m$  was different then in two first cases. But from all three courses is evidently that maximum of efficiency is in neighborhood of one third of nominal load and then efficiency declines. That's typical behavior for loaded PMSM.



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## **LIST OF APPENDIXES**

APPENDIX A: IDEAL MODEL OF PMSM

APPENDIX B: MODEL OF PMSM WITH LOSSES

APPENDIX C: SWITCHING BLOCK

APPENDIX D: ELECTRIC DRIVE WITH PMSM

APPENDIX E: TRANSFER FUNCTIONS AND CONSTANTS

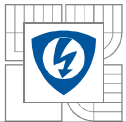
APPENDIX F: CONSTANT LOAD RESULTS

APPENDIX G: VARIABLE LOAD RESULTS

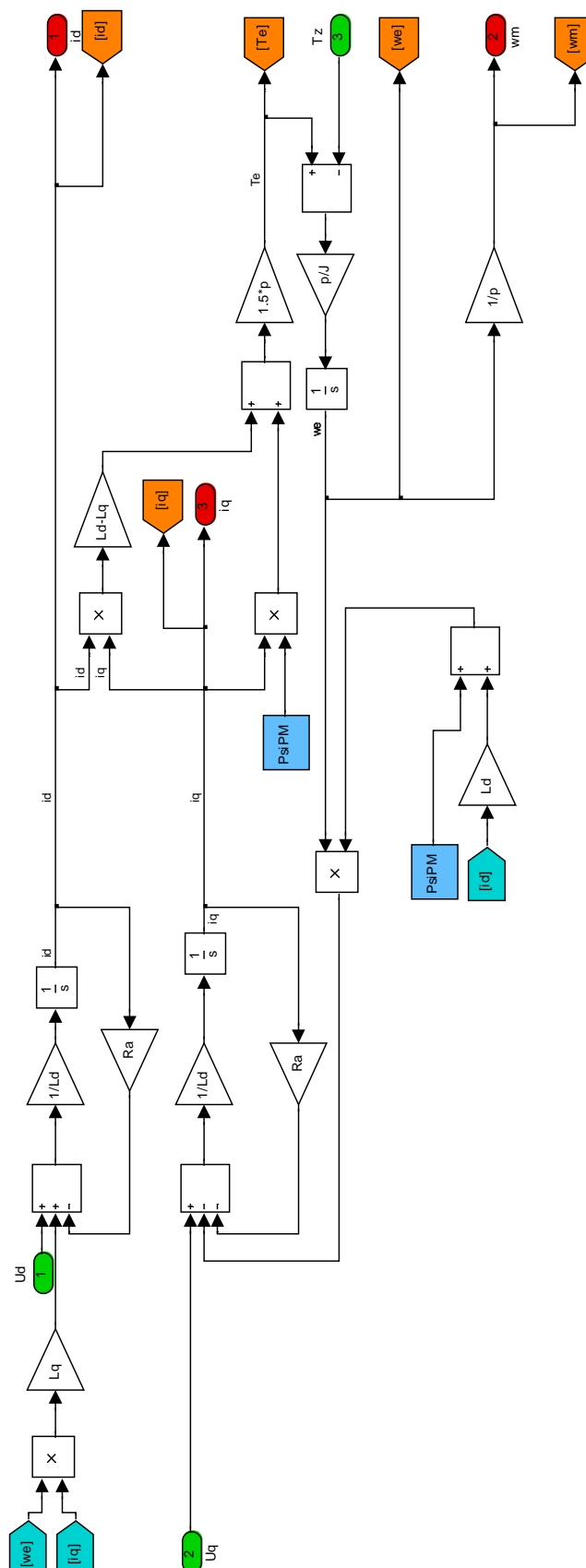
APPENDIX H: SWITCH FROM MTPA TO FW CTRL. STGY.

APPENDIX I: CONTROL STRATEGIES EFFICIENCIES

APPENDIX J: CONFIGURATION M-FILE



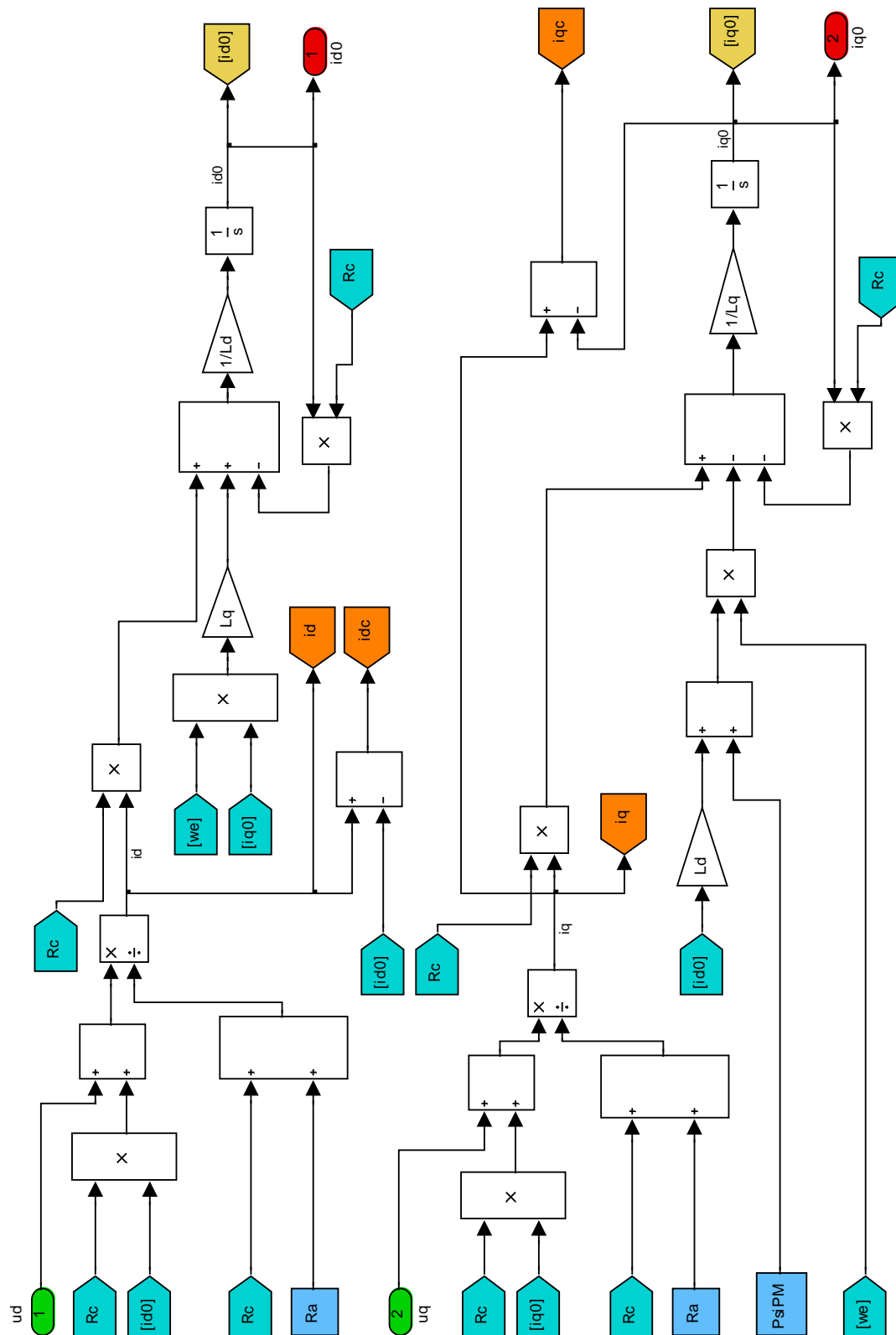
## APPENDIX A: IDEAL MODEL OF PMSM

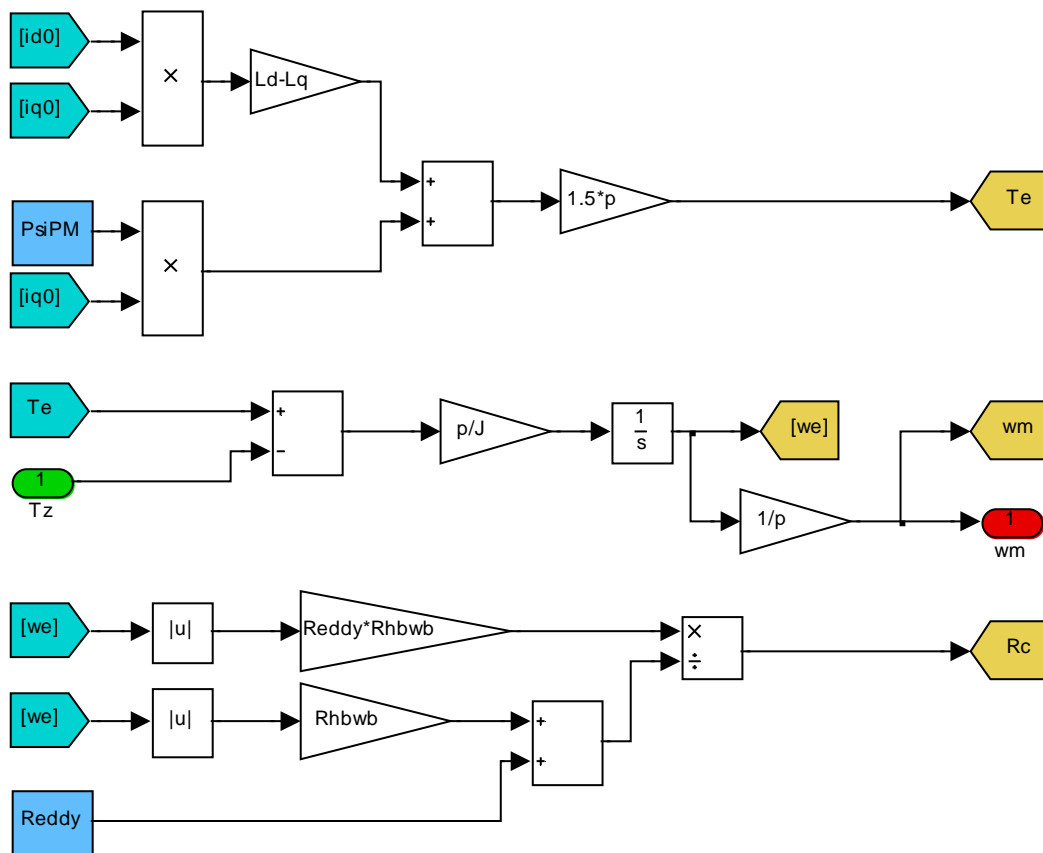




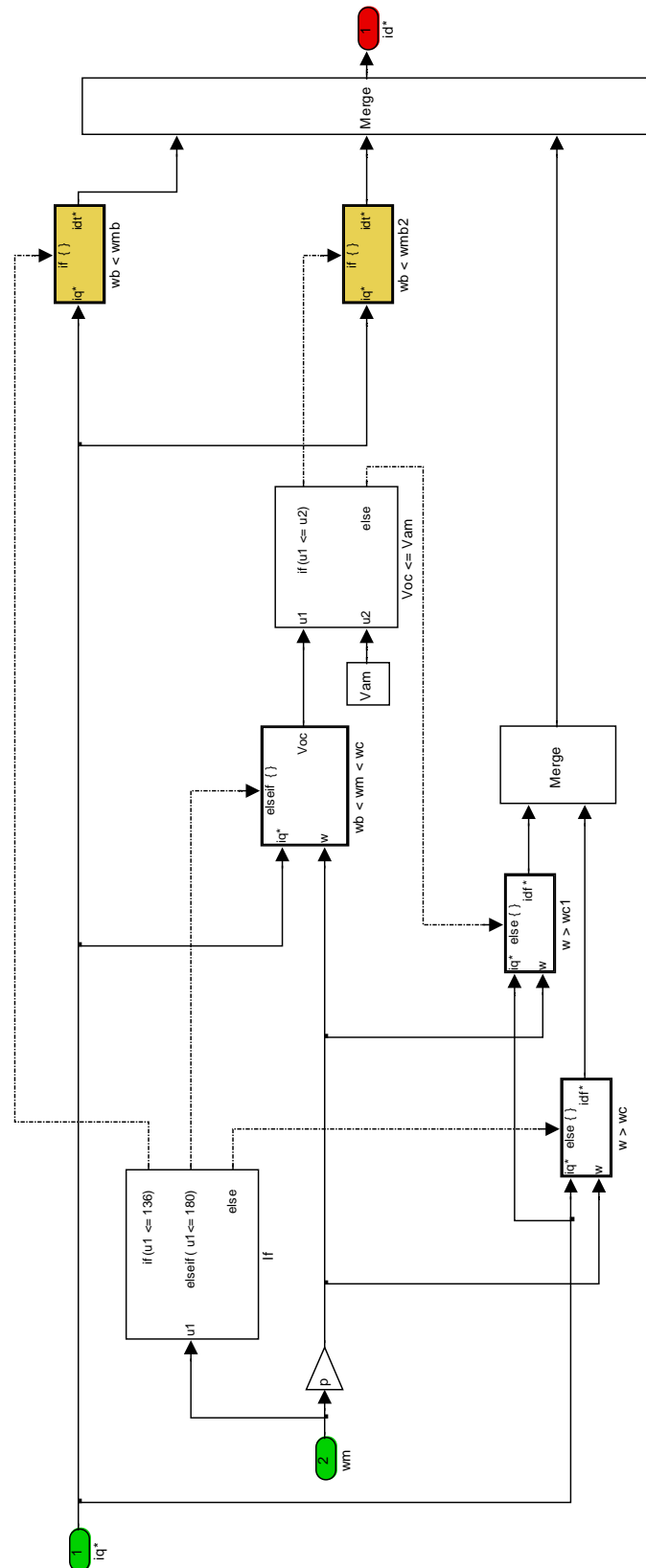


## APPENDIX B: MODEL OF PMSM WITH LOSSES

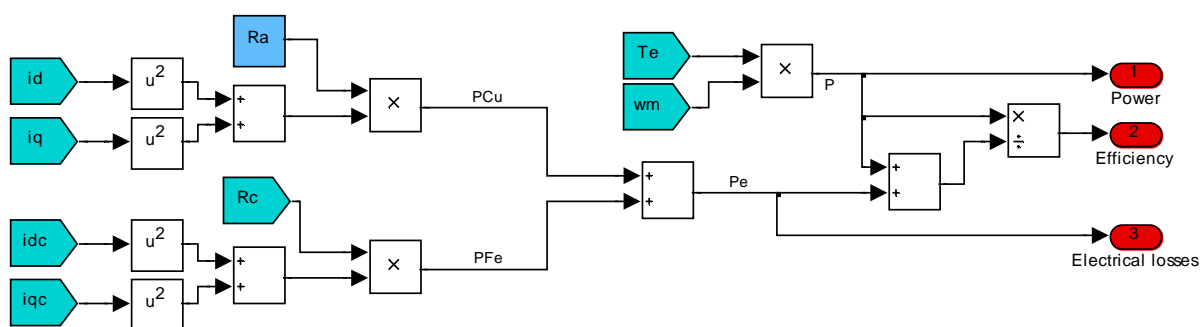
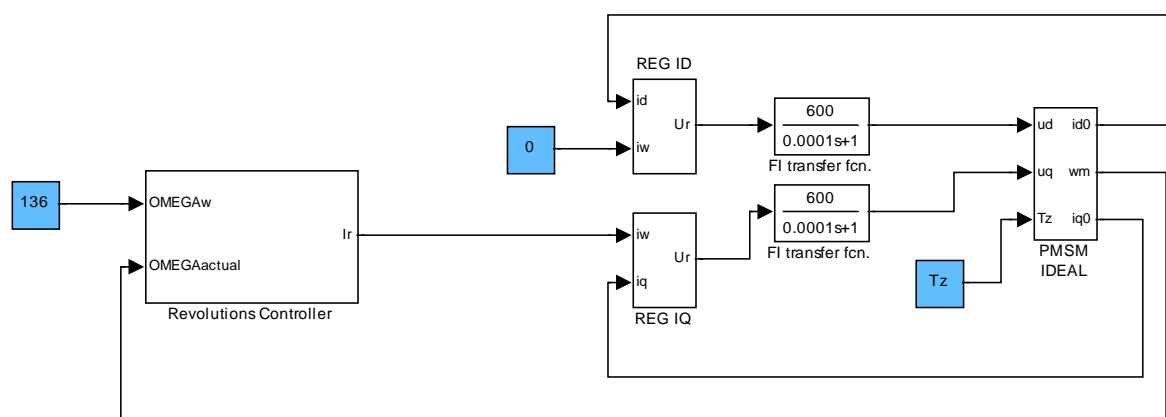




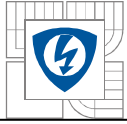
## APPENDIX C: SWITCHING BLOCK







### Efficiency calculation



## APPENDIX E: TRANSFER FUNCTIONS AND CONSTANTS

### Frequency inverter

Switching frequency of inverter is  $f=5000\text{Hz}$ . That means that time constant of inverter is:

$$\tau_{FI} = \frac{1}{2f} = \frac{1}{10000} = 0,0001s$$

another important coefficient is constant of inverter and it is given by manufacturer. For this thesis was chosen  $K_{FI}=600$ . Due this, the transfer function of frequency inverter is:

$$F_{FI} = \frac{K_{FI}}{1 + p\tau_{FI}}$$

Next transfer function is transfer function of sampler of frequency inverter. In this thesis was chosen sampler frequency  $f=10000\text{Hz}$ .

$$\tau_{SAMP} = \frac{1}{f} = \frac{1}{10000} = 0,0001s$$

### Revolutions sensor

Sampling frequency of sensor is usually about  $1000\text{Hz}$ , so time constant of this sensor is:

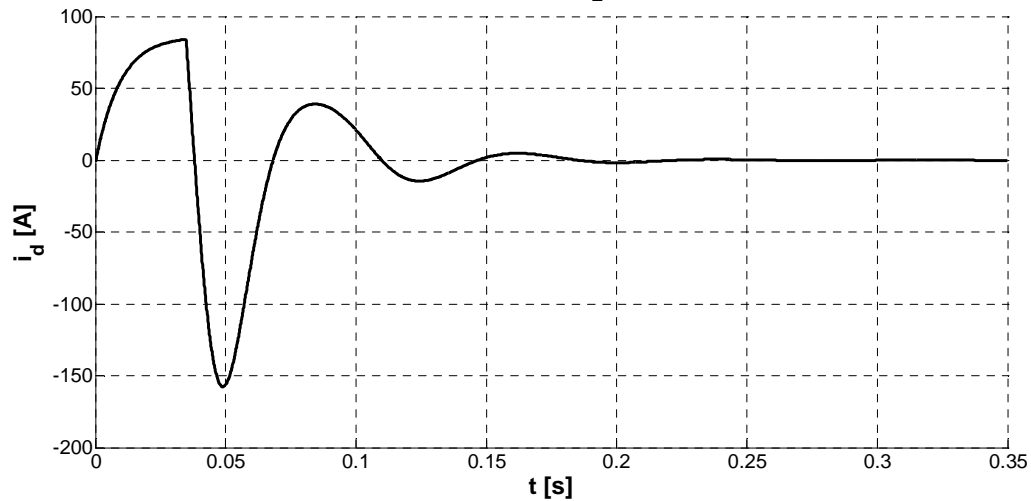
$$\tau_{SAMP2} = \frac{1}{f} = \frac{1}{1000} = 0,001s$$

and then transfer function of revolutions sampler is:

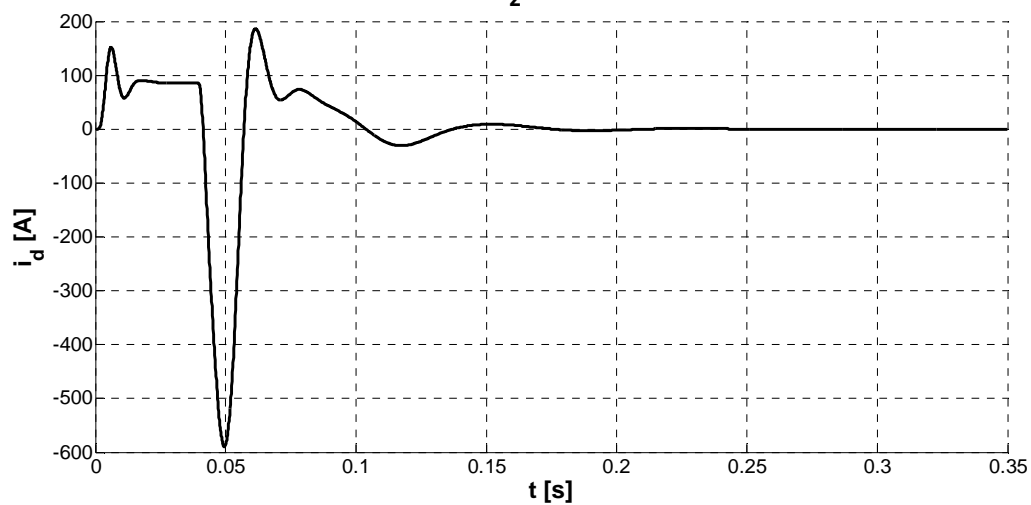
$$F_{SAMP2} = \frac{1}{1 + p\tau_{SAMP2}}$$

## APPENDIX F: CONSTANT LOAD RESULTS

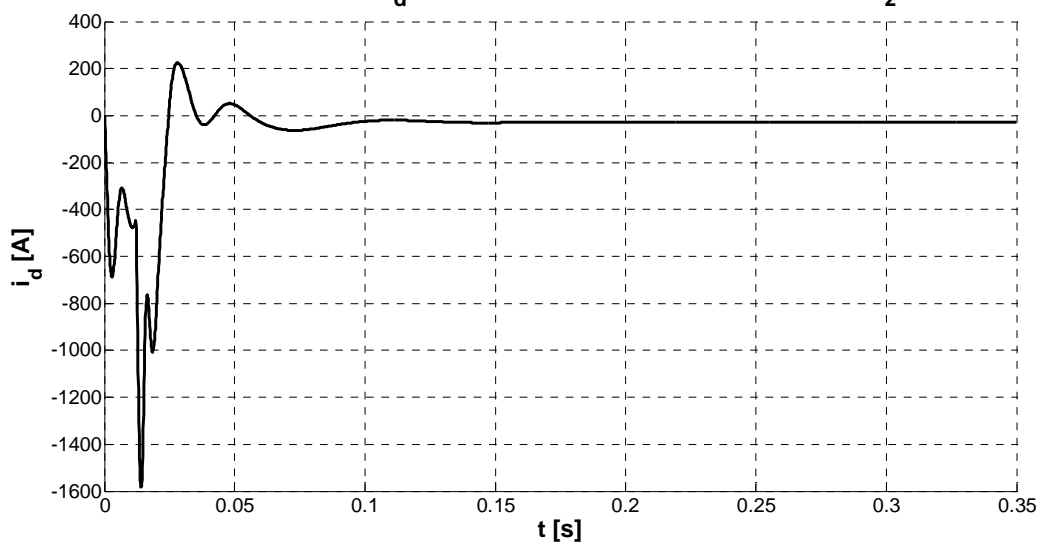
Graph of course of  $i_d$  current for  $i_d=0$  control strategy  
(No Losses) ( $T_z=\text{konst}$ )

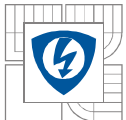


Graph of course of  $i_d$  current for  $i_d=0$  control strategy  
( $T_z=\text{konst}$ )

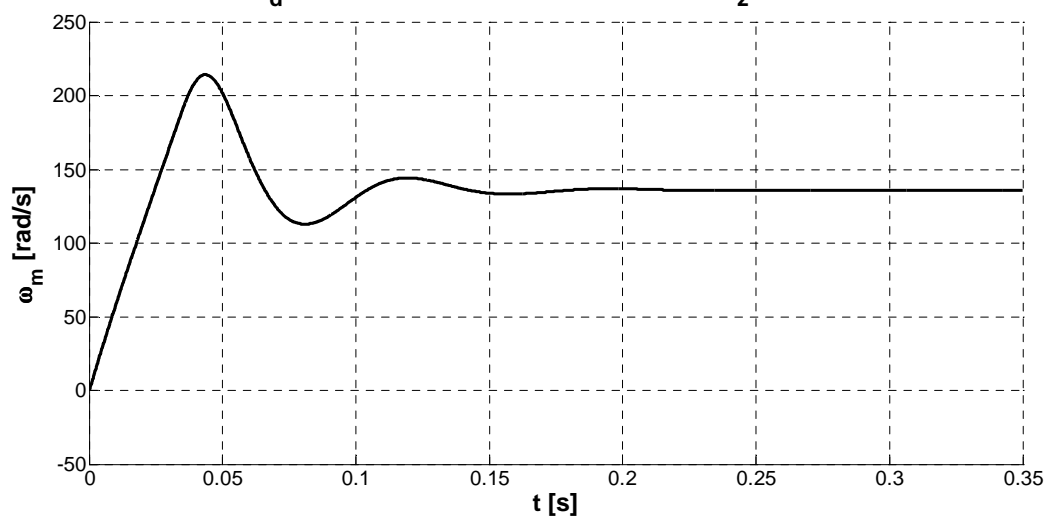


Graph of course of  $i_d$  current for MTPA control strategy ( $T_z=\text{konst}$ )

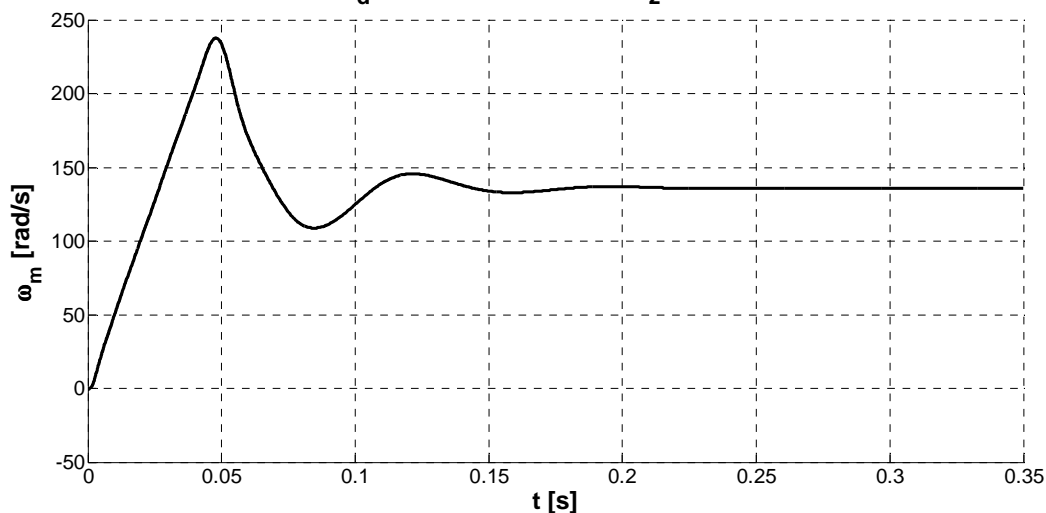




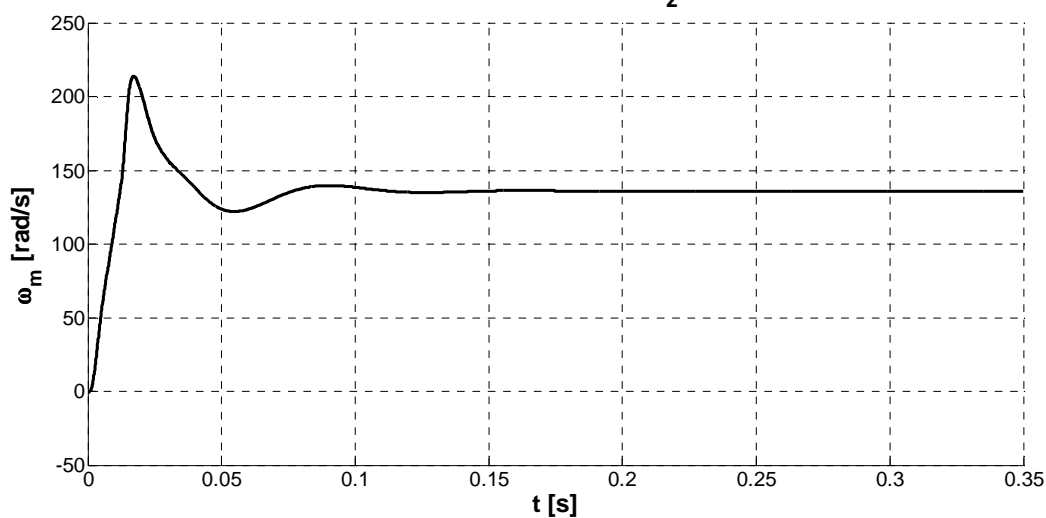
**Graph of course of  $\omega_m$**   
 **$i_d=0$  control strategy (No Losses) ( $T_z=\text{konst}$ )**



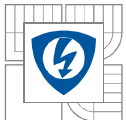
**Graph of course of  $\omega_m$**   
 **$i_d=0$  control strategy ( $T_z=\text{konst}$ )**



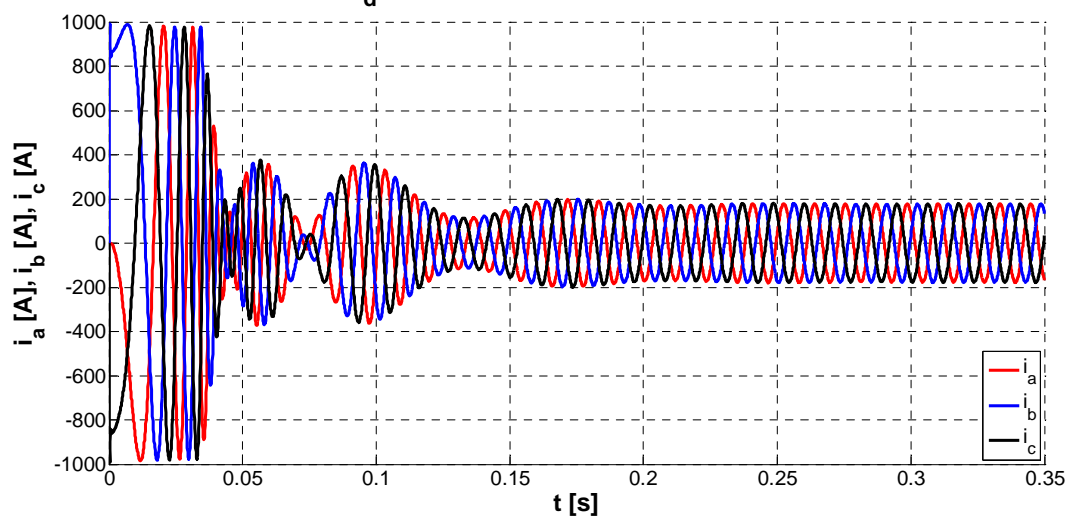
**Graph of course of  $\omega_m$**   
**MTPA control strategy ( $T_z=\text{konst}$ )**



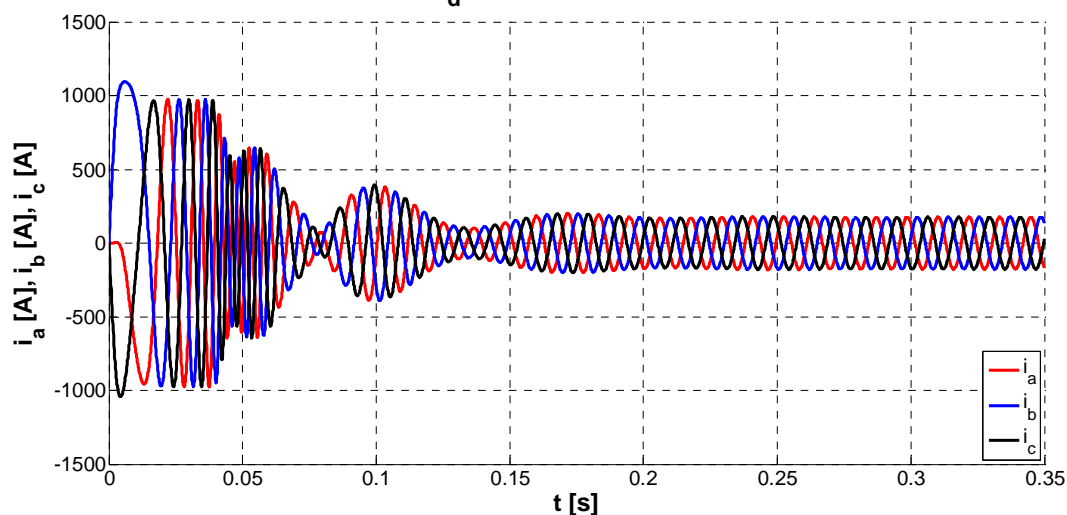




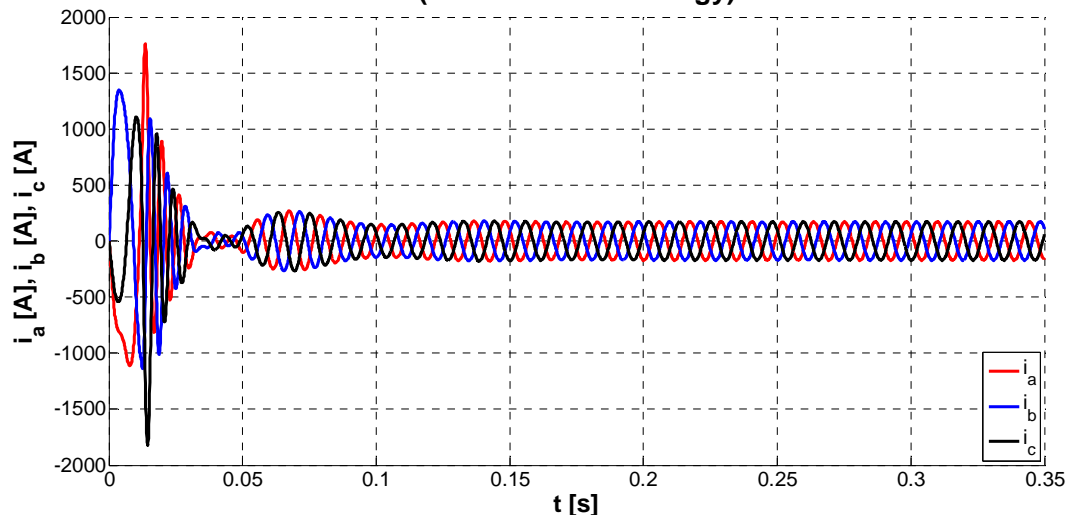
**Graph of three phase current ( $T_z = \text{konst}$ )  
( $i_d = 0$  control strategy - No Losses)**



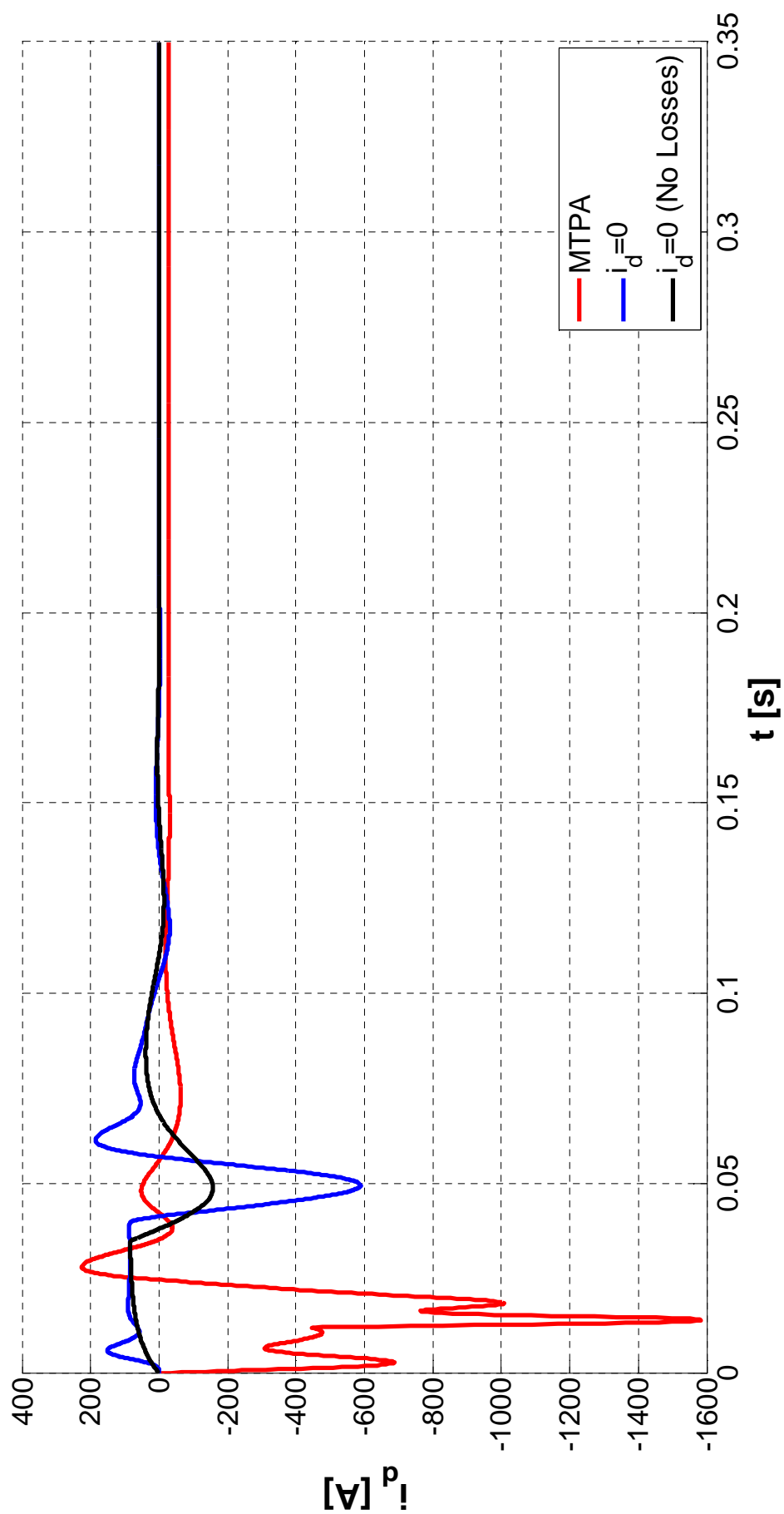
**Graph of three phase current ( $T_z = \text{konst}$ )  
( $i_d = 0$  control strategy)**



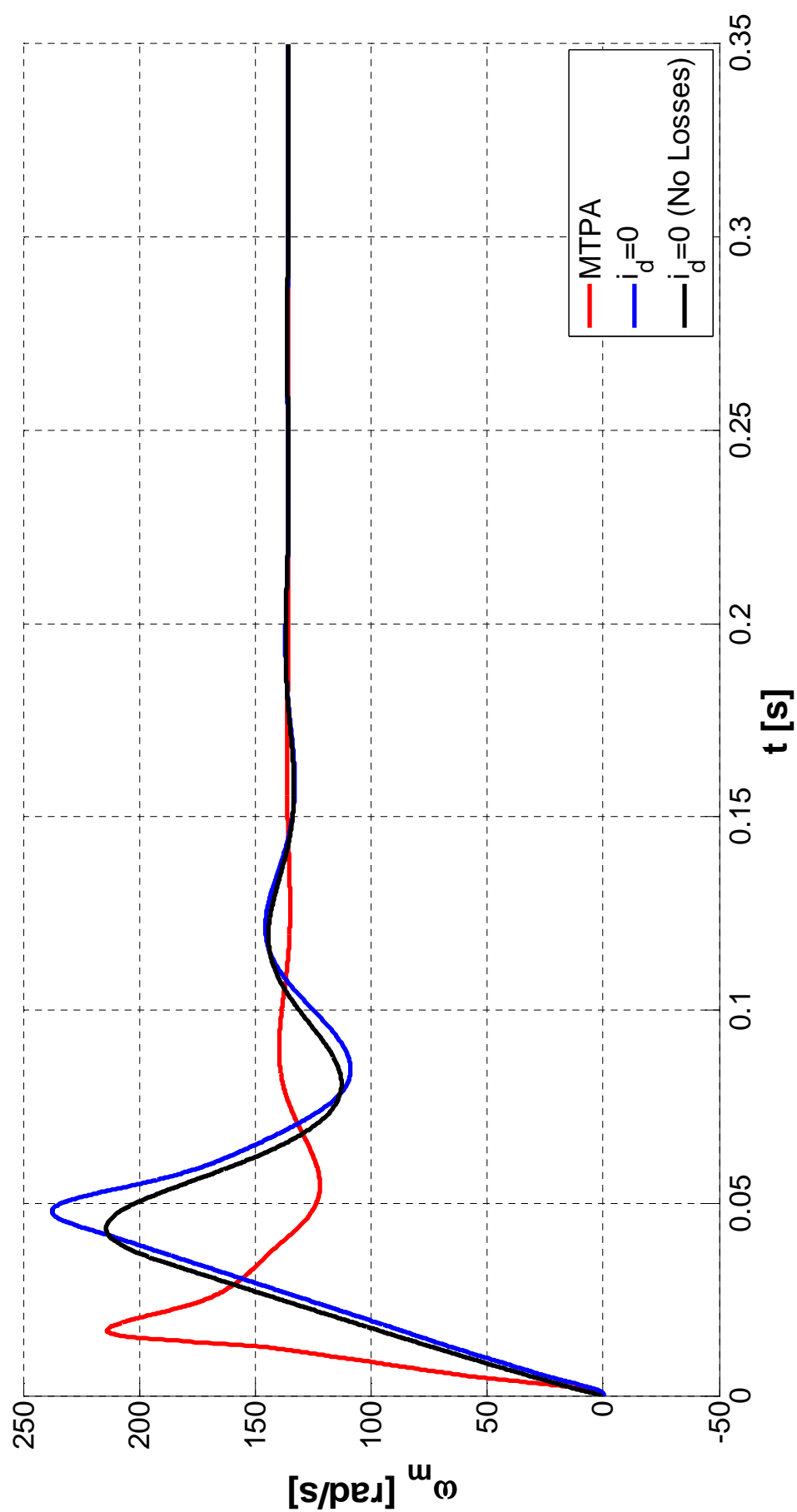
**Graph of three phase current ( $T_z = \text{konst}$ )  
(MTPA control strategy)**



Graph of course of  $i_d$  currents ( $T_z = \text{konst}$ )

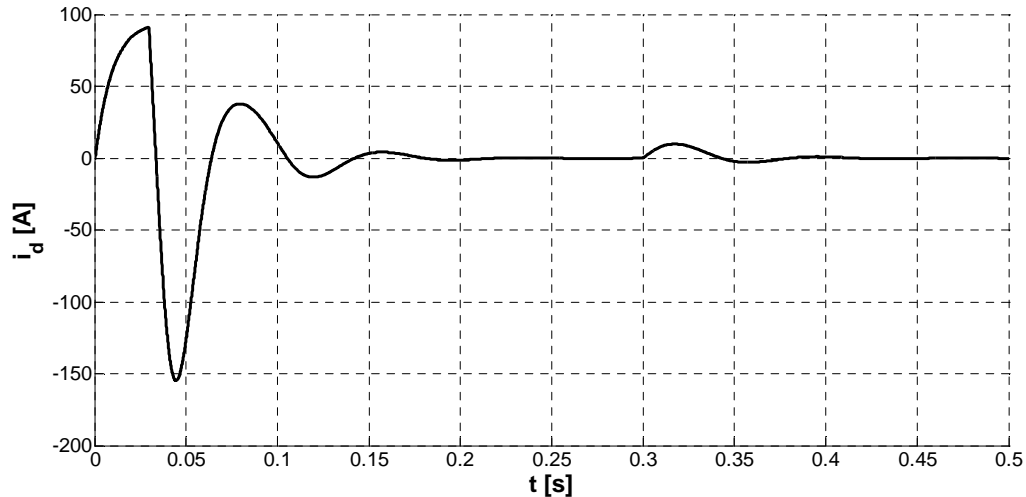


Graph of course of  $\omega_m$  ( $T_z = \text{konst}$ )

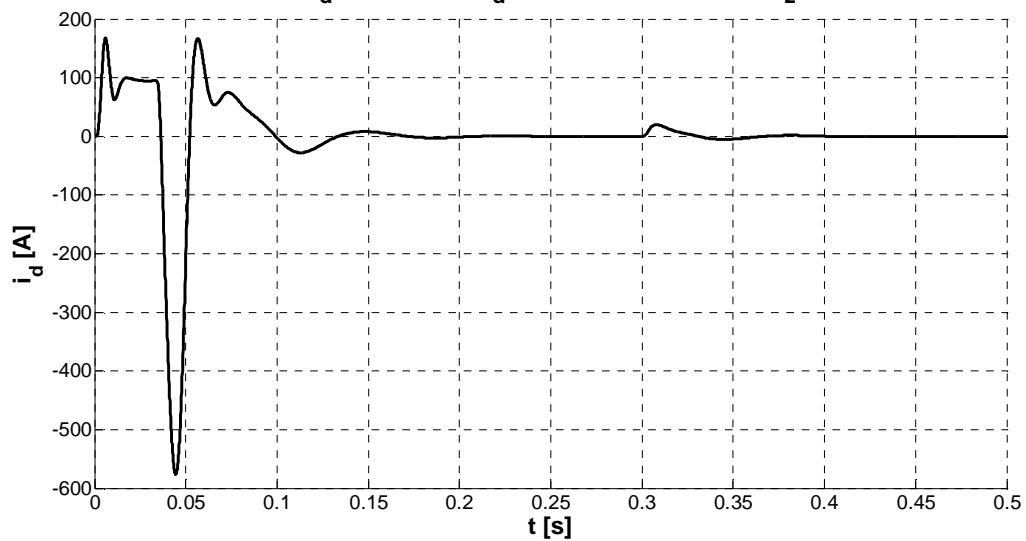


## APPENDIX G: VARIABLE LOAD RESULTS

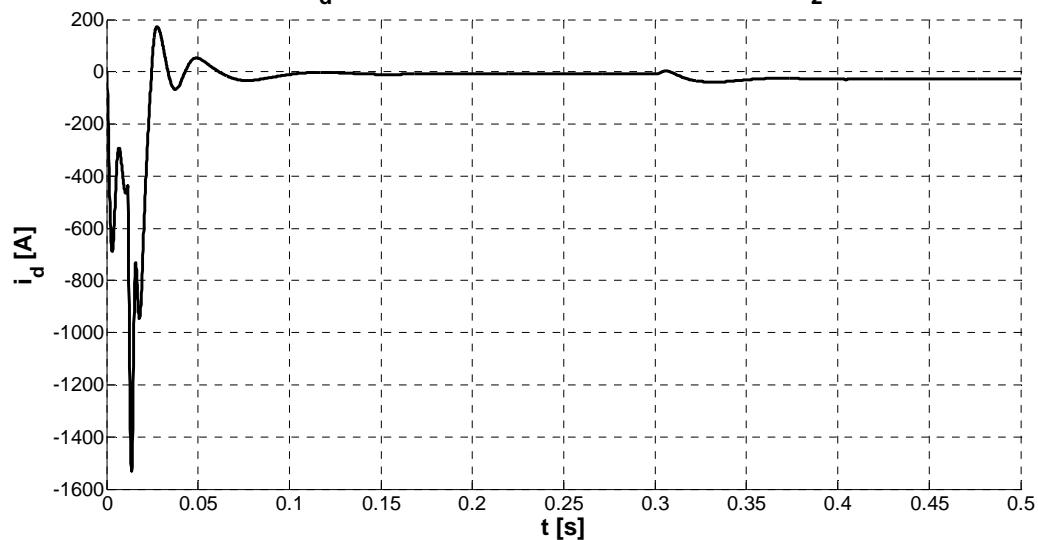
Graph of course of  $i_d$  current for  $i_d=0$  control strategy  
(No Losses) ( $T_z=100\text{Nm} \Rightarrow 200\text{Nm}$ )

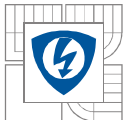


Graph of course of  $i_d$  current for  $i_d=0$  control strategy ( $T_z=100\text{Nm} \Rightarrow 200\text{Nm}$ )

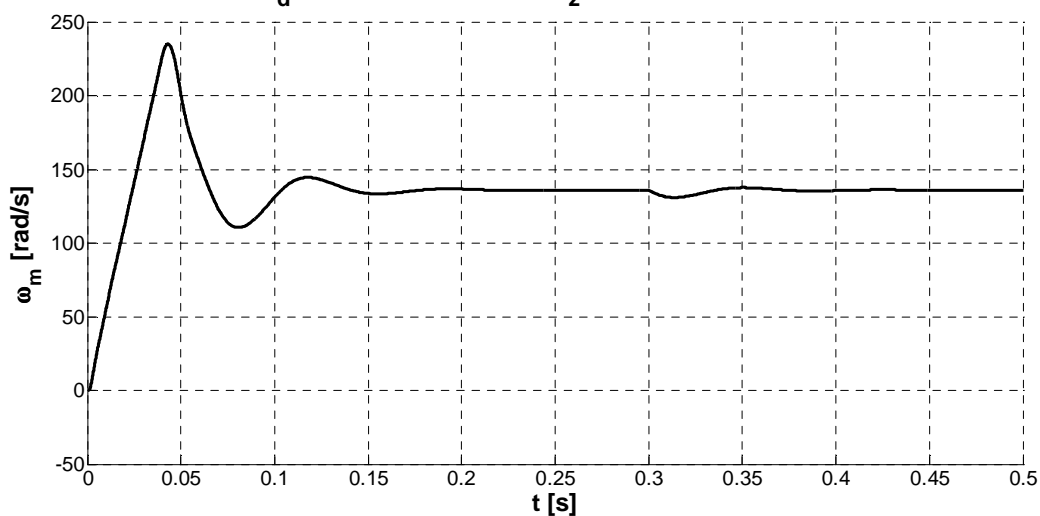


Graph of course of  $i_d$  current for MTPA control strategy ( $T_z=100\text{Nm} \Rightarrow 200\text{Nm}$ )

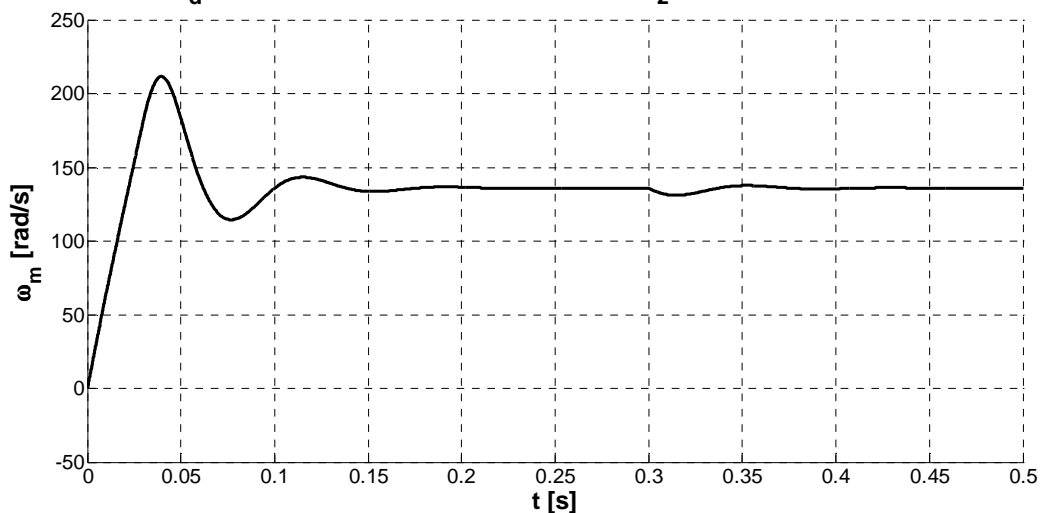




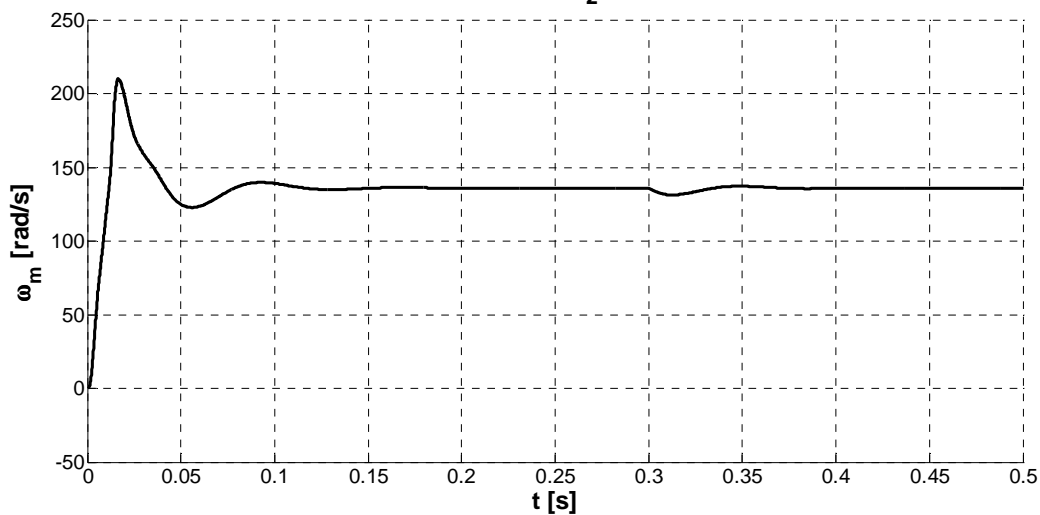
**Graph of course of  $\omega_m$**   
 **$i_d=0$  control strategy ( $T_z=100\text{Nm} \Rightarrow 200\text{Nm}$ )**

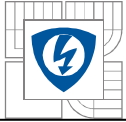


**Graph of course of  $\omega_m$**   
 **$i_d=0$  control strategy (No Losses) ( $T_z=100\text{Nm} \Rightarrow 200\text{Nm}$ )**

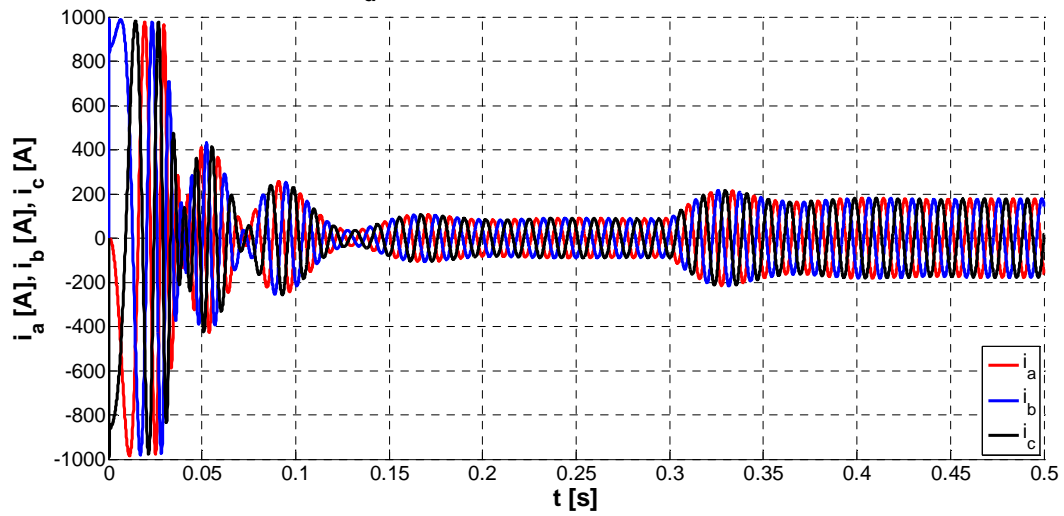


**Graph of course of  $\omega_m$**   
**MTPA control strategy ( $T_z=100\text{Nm} \Rightarrow 200\text{Nm}$ )**

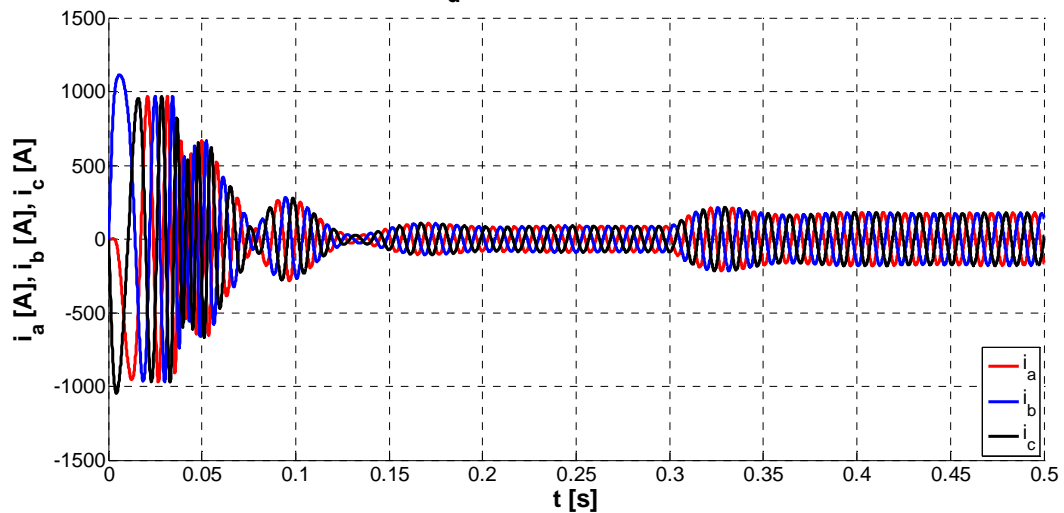




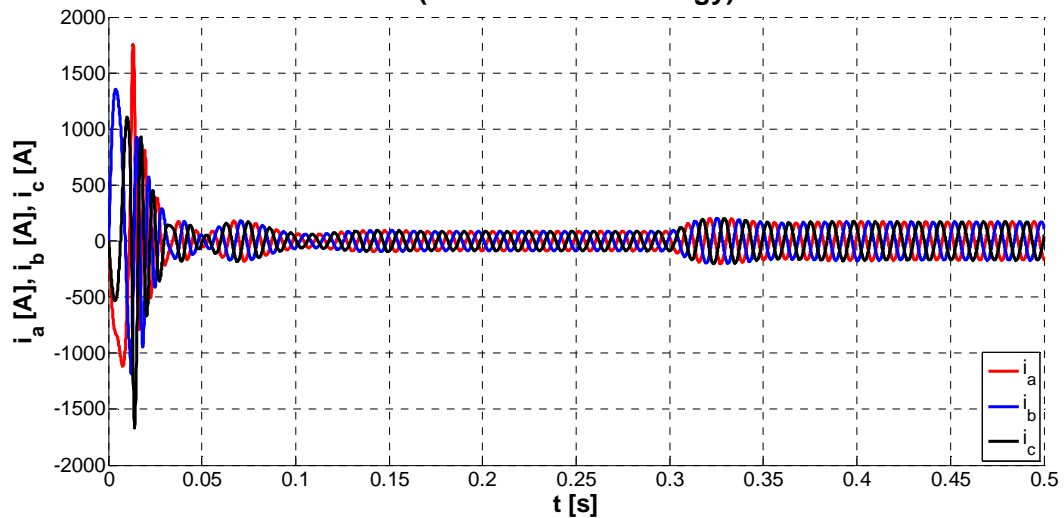
**Graph of three phase current ( $T_z=100\text{Nm} \Rightarrow 200\text{Nm}$ )  
( $i_d=0$  control strategy - No Losses)**



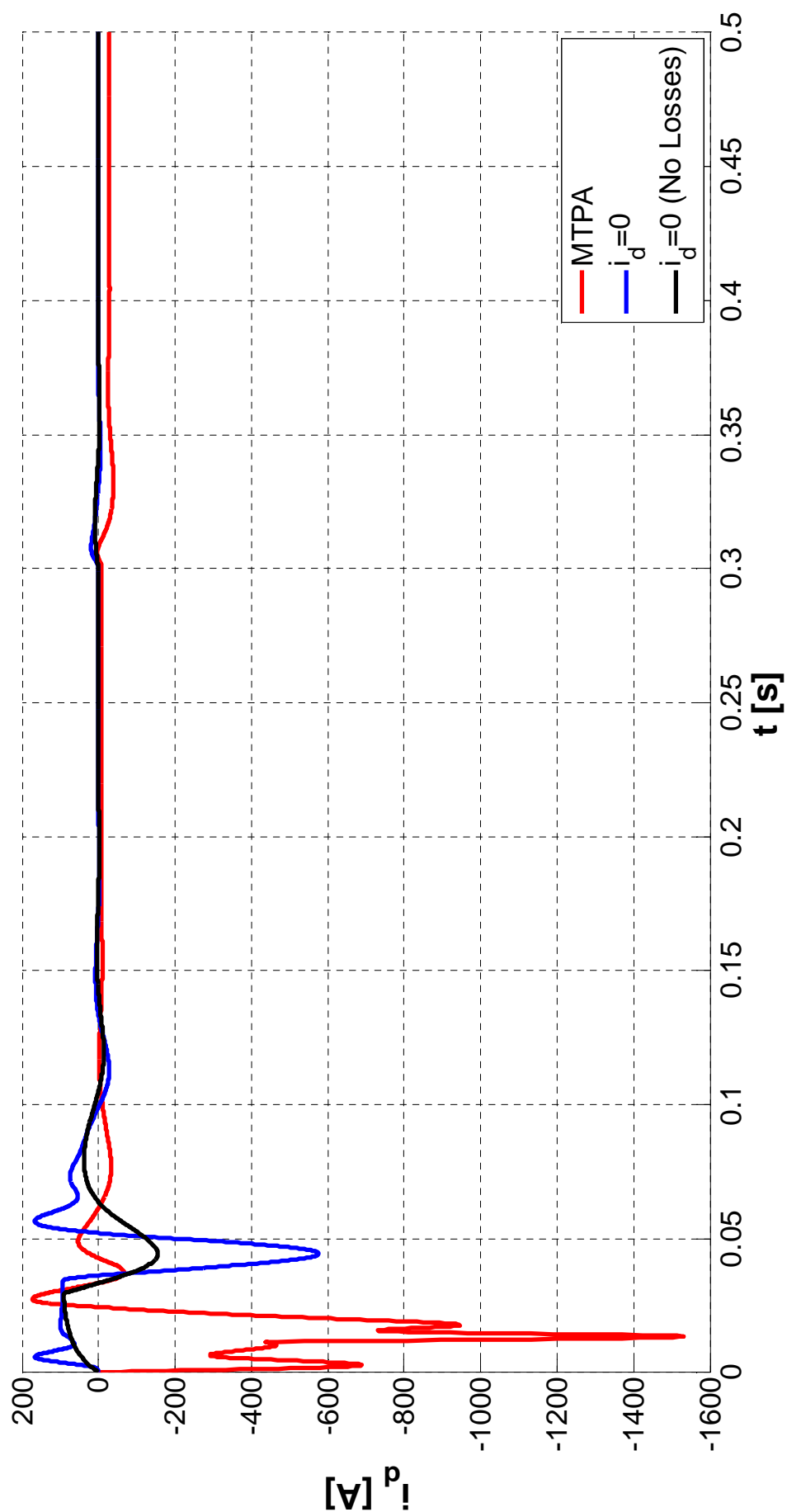
**Graph of three phase current ( $T_z=100\text{Nm} \Rightarrow 200\text{Nm}$ )  
( $i_d=0$  control strategy)**



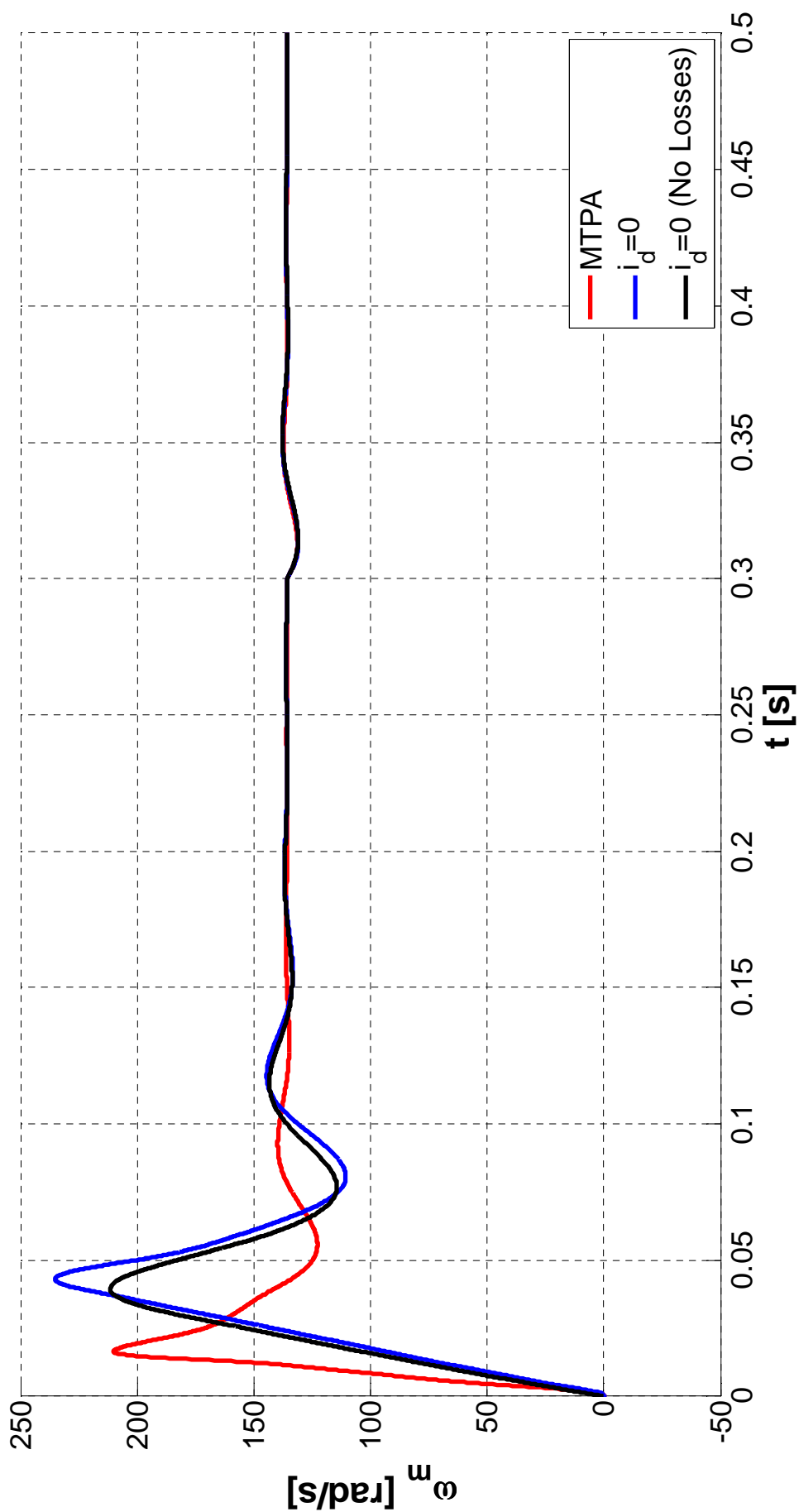
**Graph of three phase current ( $T_z=100\text{Nm} \Rightarrow 200\text{Nm}$ )  
(MTPA control strategy)**



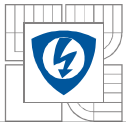
Graph of course of  $i_d$  currents ( $T_z=100\text{Nm} \Rightarrow 200\text{Nm}$ )



Graph of course of  $\omega_m$  ( $T_z=100\text{Nm} \Rightarrow 200\text{Nm}$ )

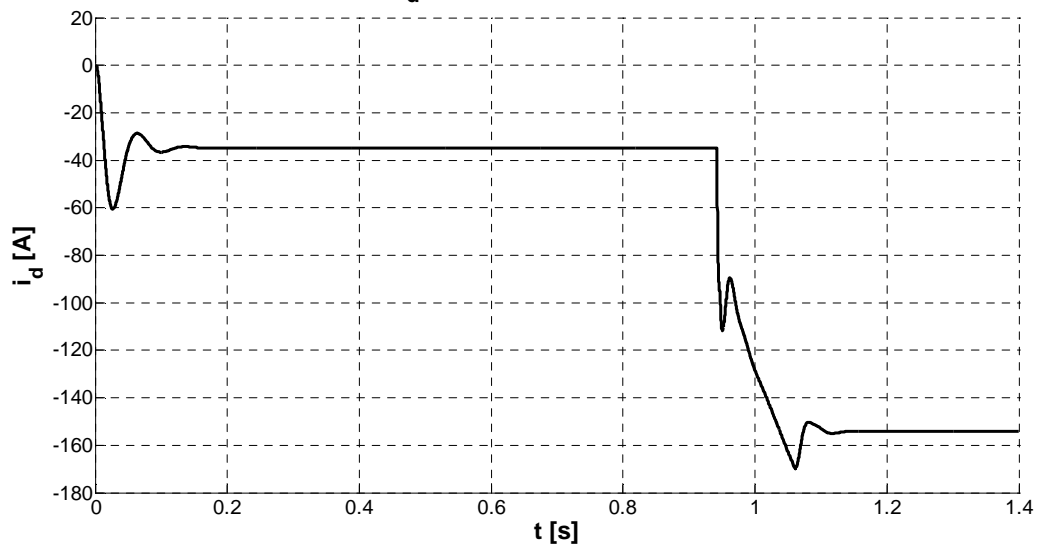




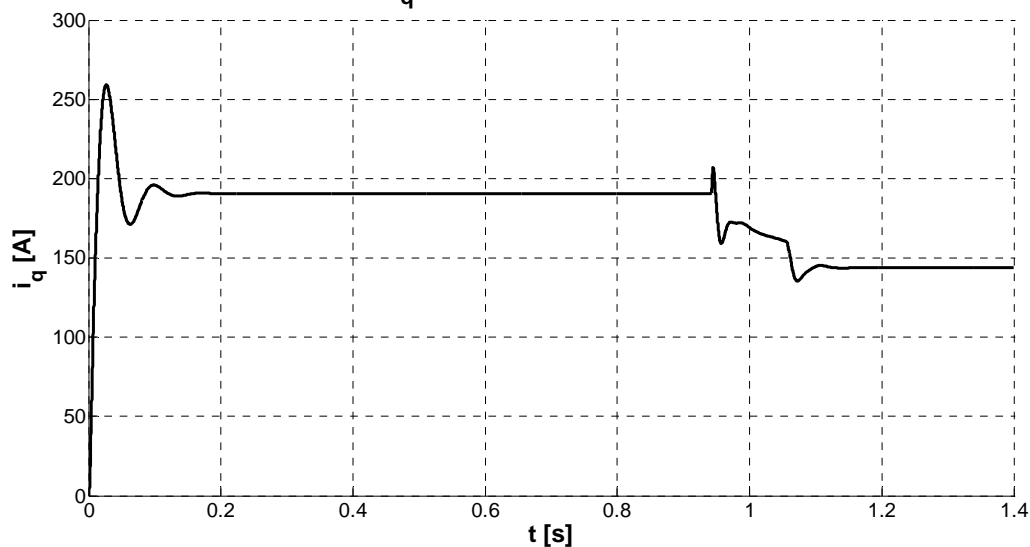


## APPENDIX H: SWITCH FROM MTPA TO FW CTRL. STGY.

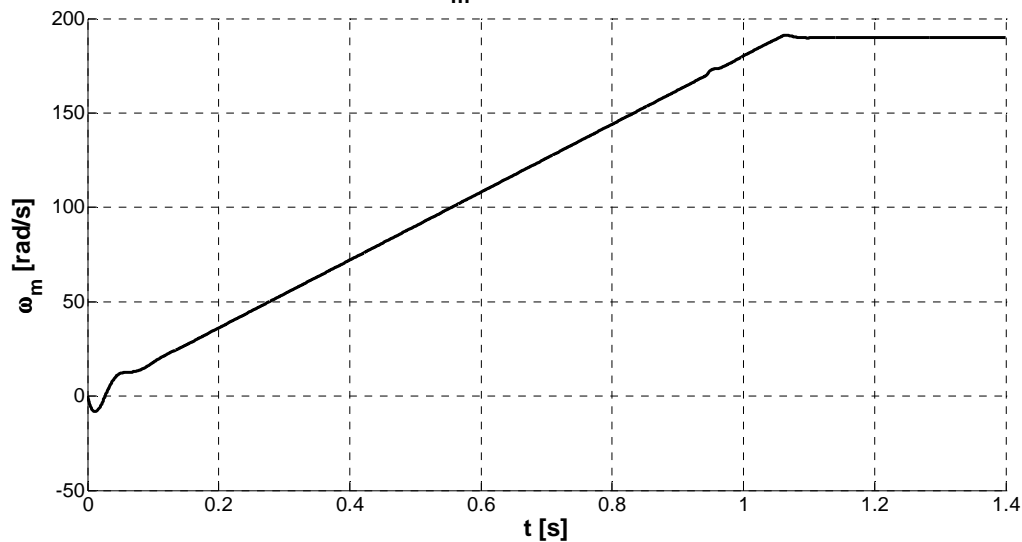
Graph of course of  $i_d$  current for MTPA and FW control strategy

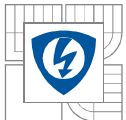


Graph of course of  $i_q$  current for MTPA and FW control strategy

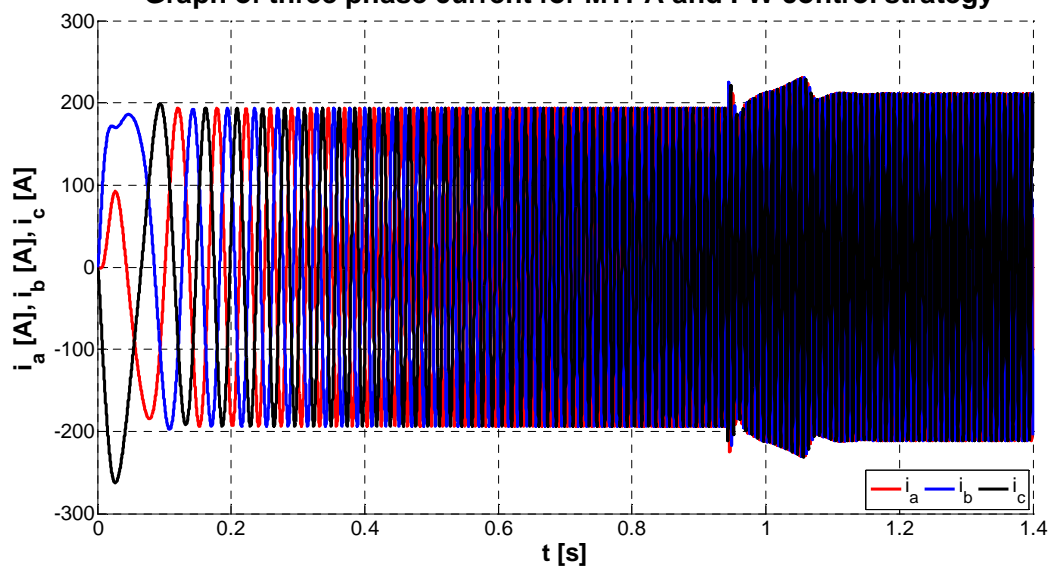


Graph of course of  $\omega_m$  for MTPA and FW control strategy

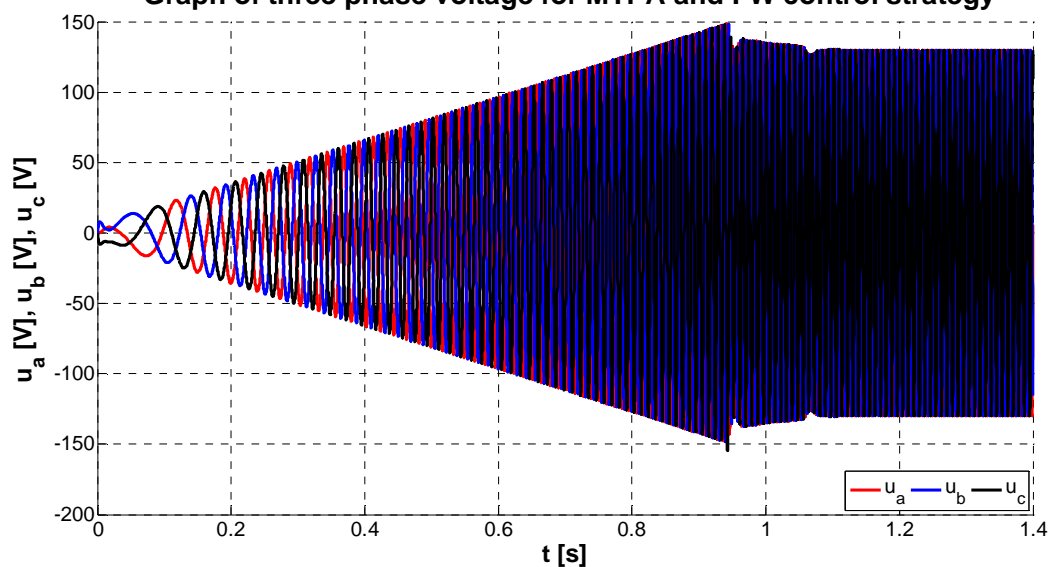




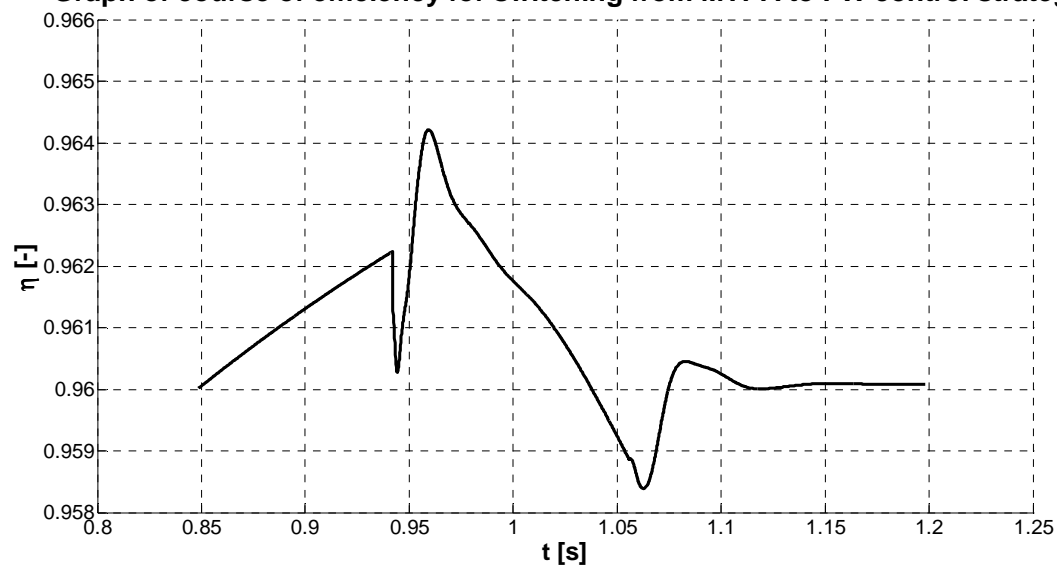
**Graph of three phase current for MTPA and FW control strategy**



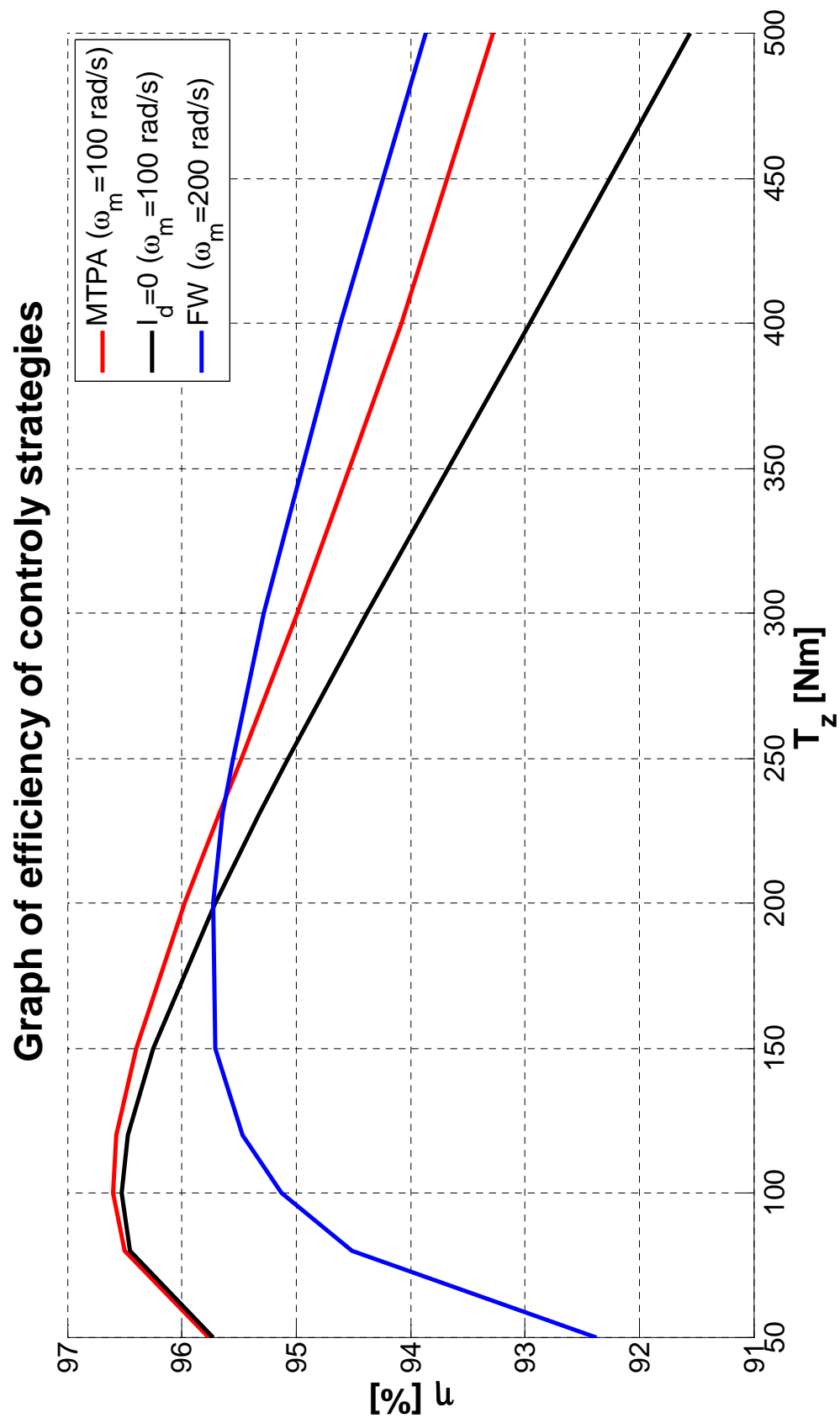
**Graph of three phase voltage for MTPA and FW control strategy**



**Graph of course of efficiency for switching from MTPA to FW control strategy**



## APPENDIX I: CONTROL STRATEGIES EFFICIENCIES





## APPENDIX J: CONFIGURATION M-FILE

```
clc; close all; clear all
```

```
Vnom=102;           %V,rms,phase
Inom=203.7;         %A,rms,phase,@ Id=0 control strategy
p=4;                %pole pairs
nb=1300;             %base revolution per minute
wb=nb/60*p*2*pi;    %base angular speed
Ra=0.0281;           %ohm, Armature resistance, phase, 165°C
PsiPM=0.1883;        %Wb, flux linkage due to Permanent Magnets
Ld=0.3268e-3;        %H,
Lq=0.6089e-3;        %H,

hb=95.73;           %ohm,Hysteresis resistance at base speed of 1300rpm
Reddy=82.21;         %ohm,Eddz current resistance
J=0.147;             %Moment of inertia of the entire rotor assembly
Rhbw=Rhb/544.5427 %Rh,b/we,b
```

```
%Core loss resistance in dq equivalent circuit is a parallel combination of
%Rh,b and Reddy
%Rh is directly proportional to speed: Rh=Rh,b*we/we,base , where we
%is electrical angular velocity at nbase=1300rpm
%Reddy is constant, independent on speed
```

```
Vam=200             %Max frequency inverter voltage
Iam=400              %Max frequency inverter current
```

```
fFI=5000;           %switching frequency of inverter
fSAMP=10000;         %frequency of inverter sampling
```

```
tauFI=1/(2*fFI)      %time constant of frequency inverter
tauSAMP=1/fSAMP        %time constant of sampling
tauSig=tauFI+tauSAMP  %sum constant
```

```
kFI=600/1;           %constant of frequency inverter
```

```
tauQ=Lq/Ra           %time constant of q axis
```

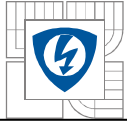
```
tau0=2*tauSig*kFI/Ra
taulq=tauQ
```

```
tauD=Ld/Ra           %time constant of d axis
tauld=tauD
```

```
tauA=2*tauSAMP*kFI/Ra %
```

```
tauSAMP=0.0008        %%%%%%
tauSAMP2=0.001         %
tauSIG=2*tauSAMP        %Time constants for controller design
tauB=12*6*PsiPM*p*tauSIG^2/J %
%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
Rc=(Rhb*Reddy)/(Rhb+Reddy) %Resistance represents copper losses
tauqLosses=Lq/(0.2*Ra*Rc) %
```



```
taudLosses=Ld/(0.2*Ra*Rc) %  
tauX=tauA %  
Pqi=(tauFI+tauqLosses)/tauX %  
Iqi=1/tauX % q regulator components with losses  
Dqi=tauqLosses*tauFI/tauX %  
  
Pdi=(tauFI+taudLosses)/tauX %  
Idi=1/tauX % d regulator components with losses  
Ddi=taudLosses*tauFI/tauX %  
  
Vom=Vam-Ra*Iam % available voltage from inverter  
PsiA=3/2*PsiPM % phase flux linkage due to PM
```